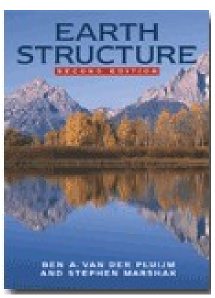
GEO-310 Lecture 4
Deformation, Strain, and Rheology

Cross-cutting tectonic veins in Moore’s Creek, Lambertville, NJ



Earth Structure (2nd Edition), 2004
W.W. Norton & Co, New York
Many slides by Ben van der Pluijm

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Deformation is:

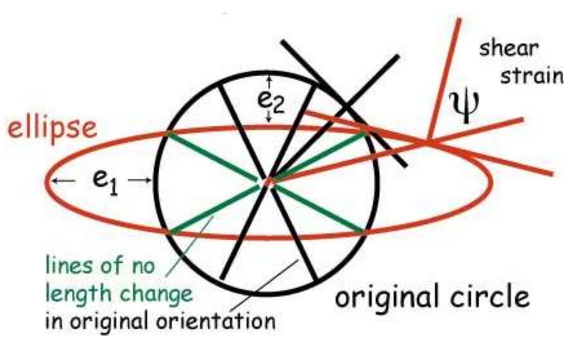
b) Strain (distortion)
- Extension (or stretch) length changes
- Internal rotation (vorticity)

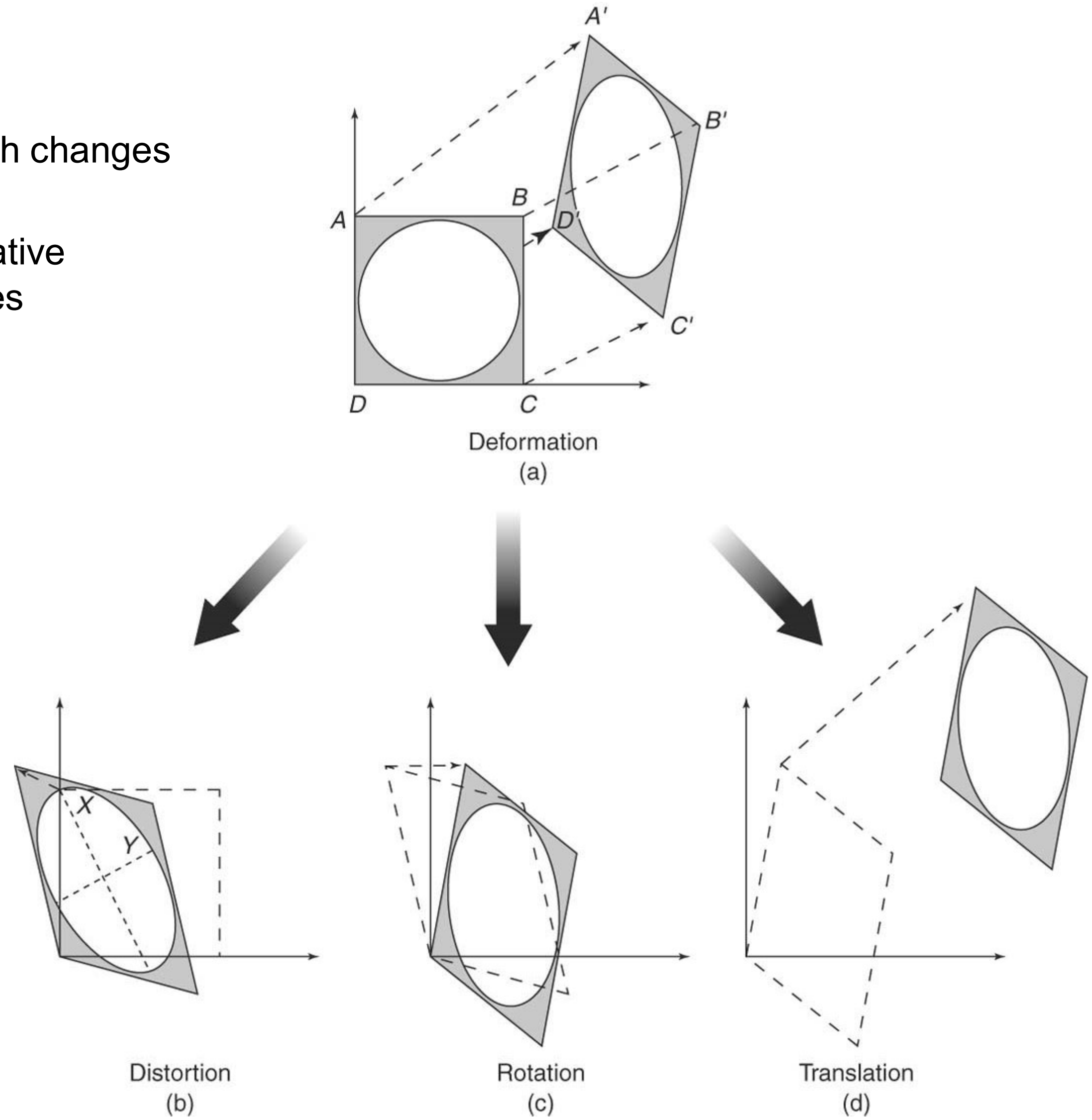
  finite strain axes rotate relative
  to instantaneous strain axes
- Volume change

c) Rigid-body rotation (or spin)

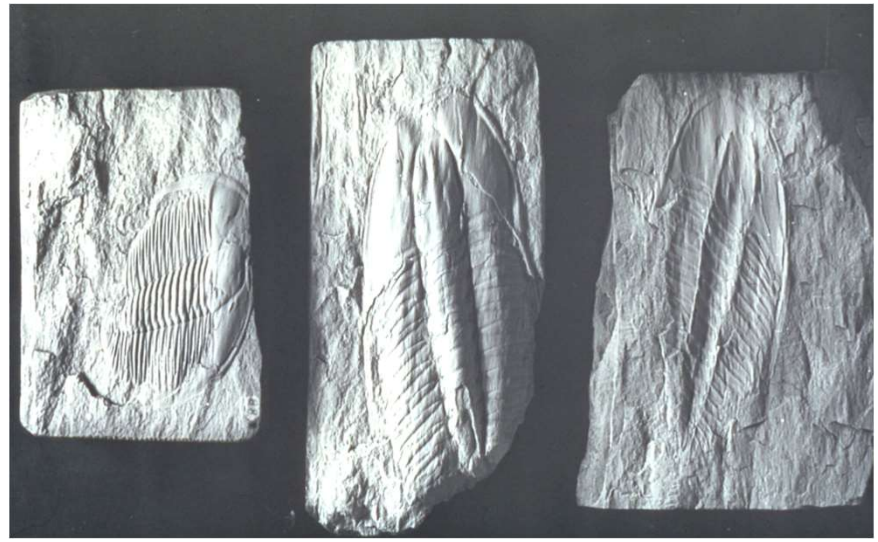
  instantaneous axes and
  finite axes rotate together

d) Rigid-body translation





Strain (distortion)



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Homogeneous vs. heterogeneous strain

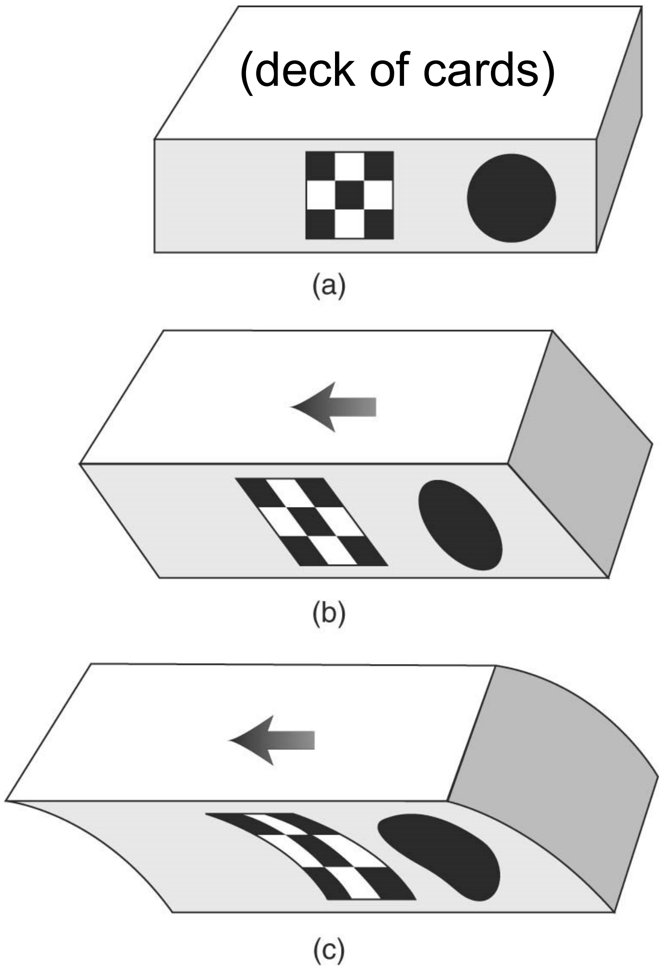


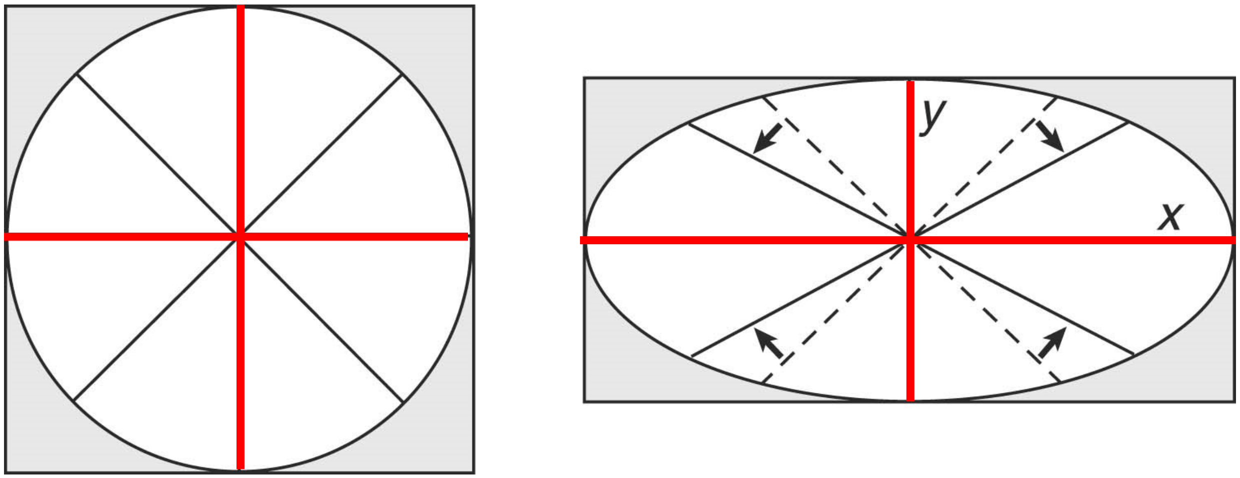
FIGURE 4.3 A square and a circle drawn on a stack of cards

Homogeneous strain (b) transform into a parallelogram and an ellipse

- Straight lines remain straight
- Parallel lines remain parallel
- Circles become ellipses
  (or spheres become ellipsoids)

Heterogeneous strain (c) is produced by variable slip on the cards, for example by increasing the slip on individual cards from bottom to top.

Principal strain axes



Homogeneous strain: transformation of square to rectangle, circle to ellipse.

- Two material lines perpendicular before and after strain are the **principal axes** of the strain ellipse (red lines).
- The dashed lines are other **material lines** that do not remain perpendicular after strain; they rotate toward long axis of strain ellipse.

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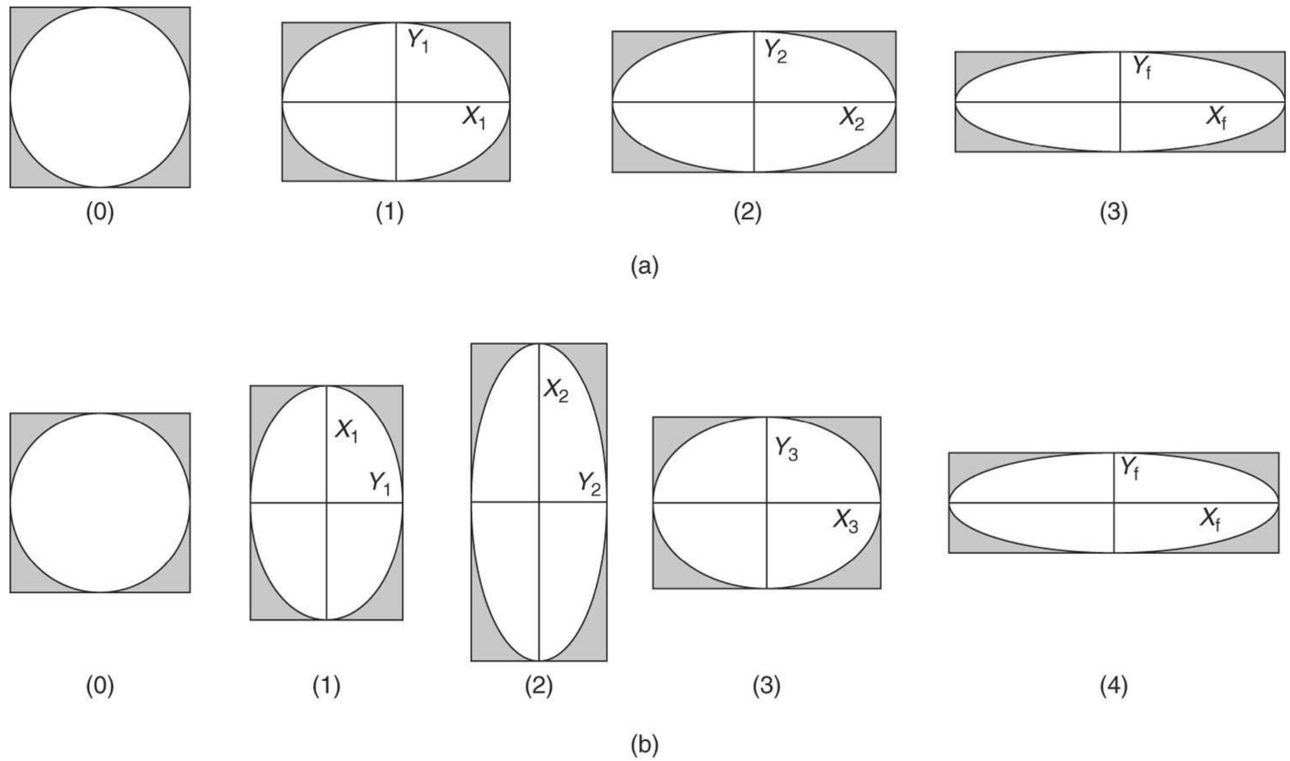
Strain path

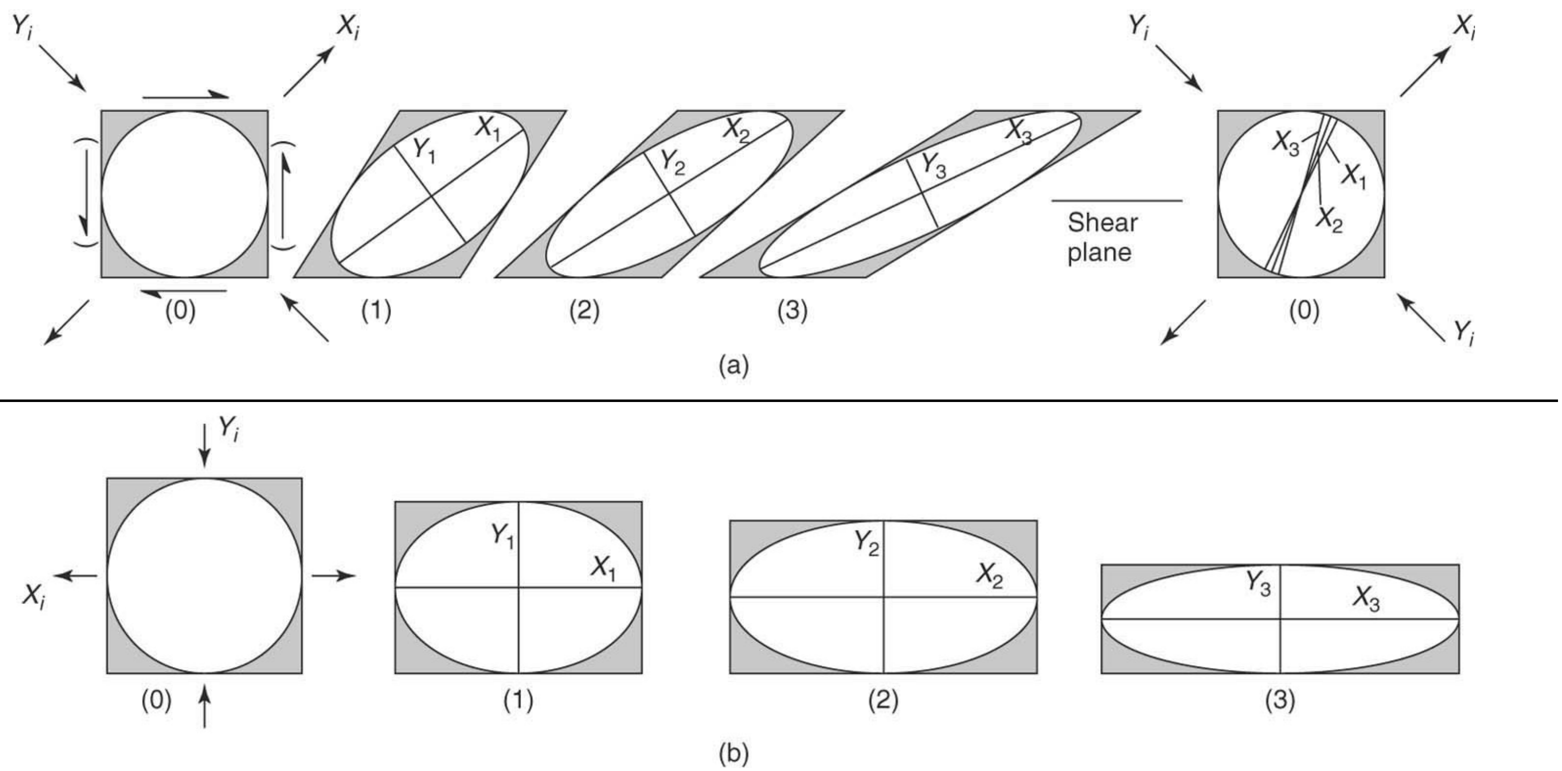
FIGURE 4.5 The finite strains, Xf and Yf, in (a) and (b) are the same, but the strain path by which each was reached is different. This illustrates the importance of understanding the incremental strain history (here, Xi and Yi) of rocks and regions and inherent limitation of finite strain analysis.



- Incremental strain (strain steps)
- Finite strain (difference between unstrained and final shape)
- (Infinitesimal strain)

Non coaxial and coaxial strain

Progressive. simple shear or non-coaxial strain



Progressive. pure shear, or coaxial strain

- The incremental strain axes are different material lines during non-coaxial strain
- In coaxial strain the incremental strain axes are parallel to the same material lines.
    - Note that the magnitude of the strain axes changes with each step.

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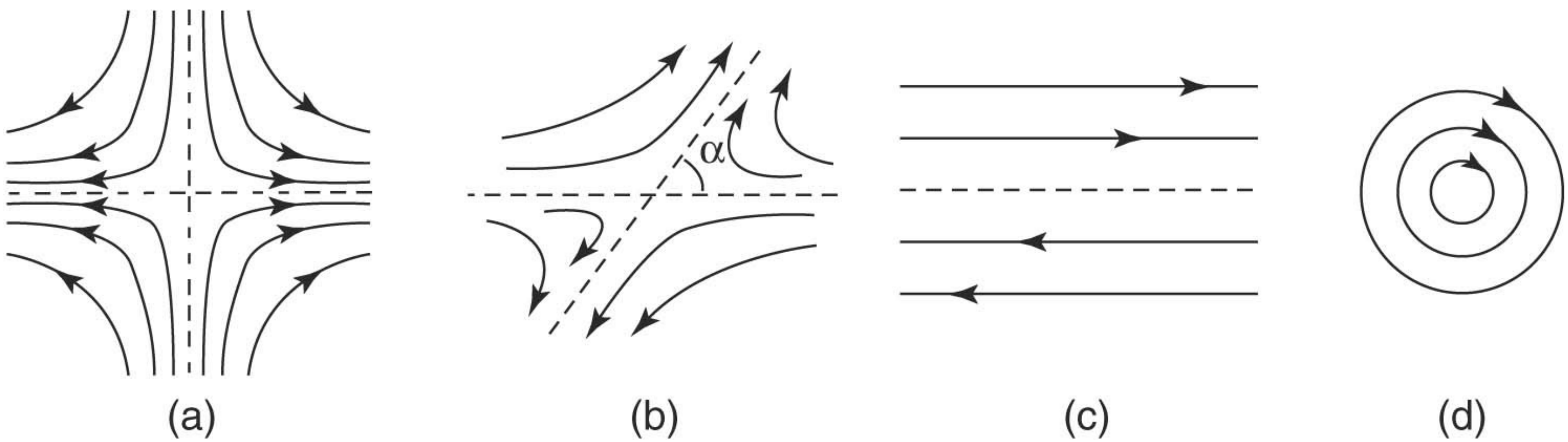
Vorticity (internal rotation) and Particle paths



general shear simple shear pure shear rigid-body rotation

FIGURE 4.8 Particle paths or flow lines during progressive strain accumulation.

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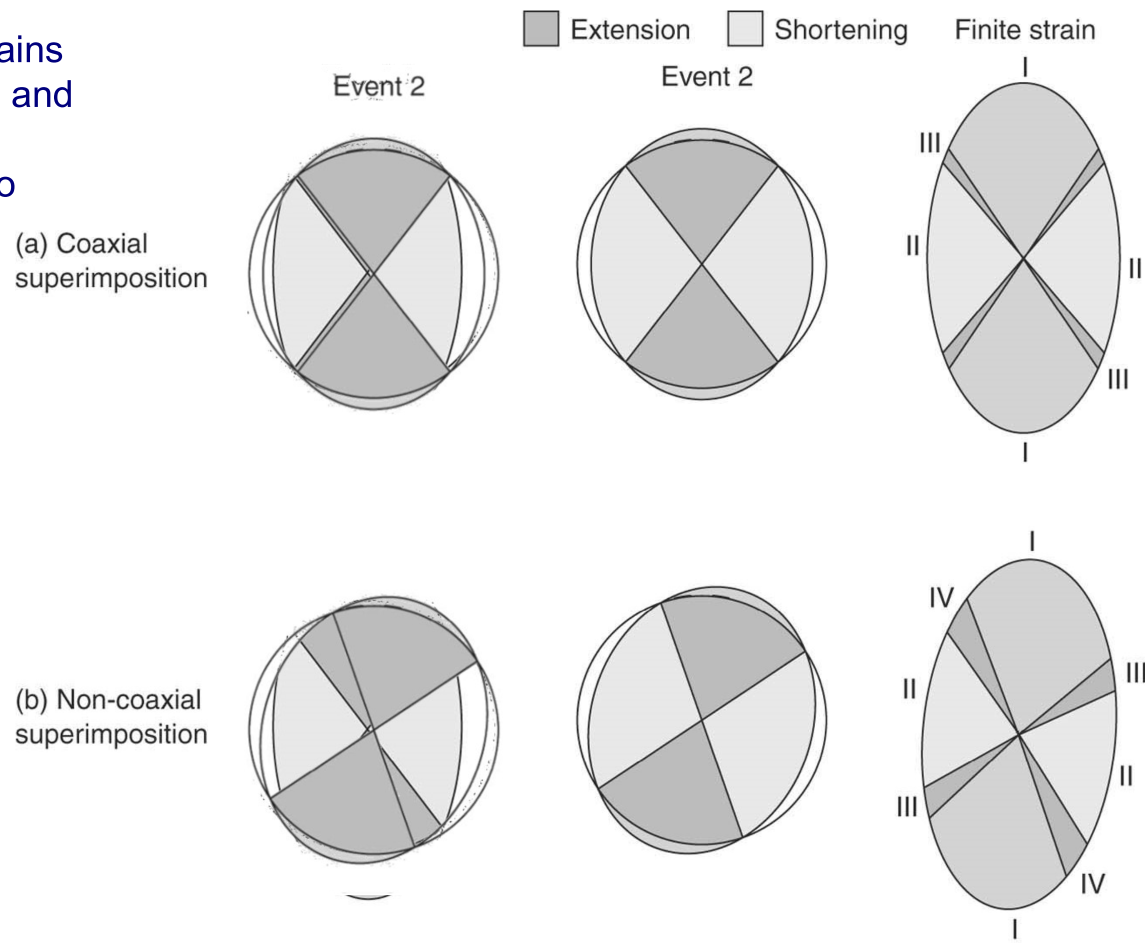
Superimposed strain

FIGURE 4.9 The strain ellipse contains regions in which lines are extended and regions where lines are shortened, which are separated by lines of zero finite elongation.

(a) Coaxial superimposition of a strain event 2 produces regions in which lines continue to be extended (I), and regions where lines continue to be shortened (II), separated by regions where shortening during event 1 is followed by extension during event 2 (III)

(b) Non-coaxial superimposition additionally produces a region where extension is followed by shortening during event 2 (IV).



Practically this means that structures reflecting different strain states may be formed, especially in non-coaxial strain regimes such as shear zones.

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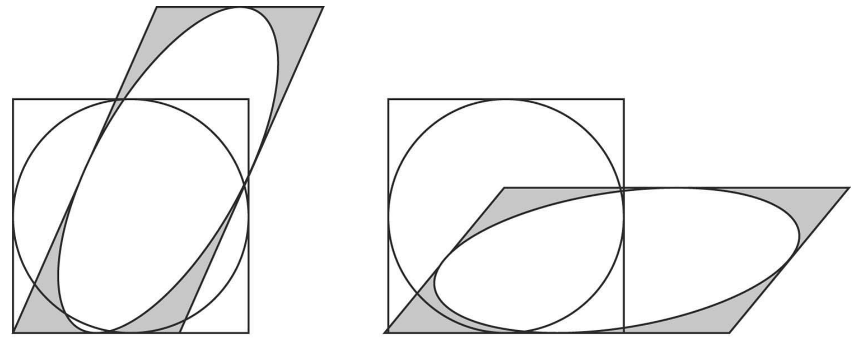
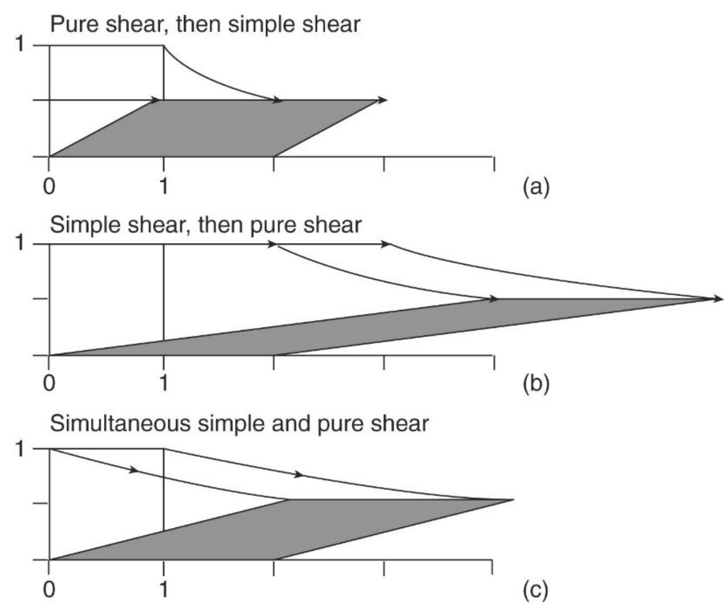
Transtension and Transpression

FIGURE 4 . 7 A combination of simple shear (a special case of noncoaxial strain) and pure shear (coaxial strain) is called *general shear* or general non-coaxial strain.

- Two types of general shear are (a) transtension and (b) transpression, reflecting extension and shortening components.



(a) transtension (b) transpression



Combinations of simple shear (non-coaxial strain) and pure shear (coaxial strain)

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Strain quantification

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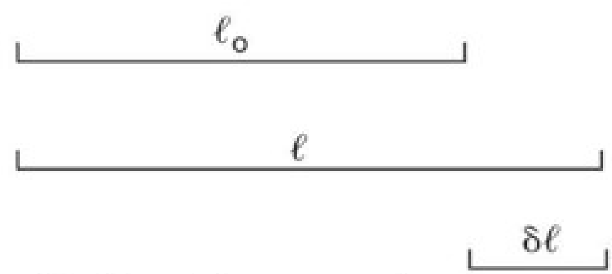
Strain quantities

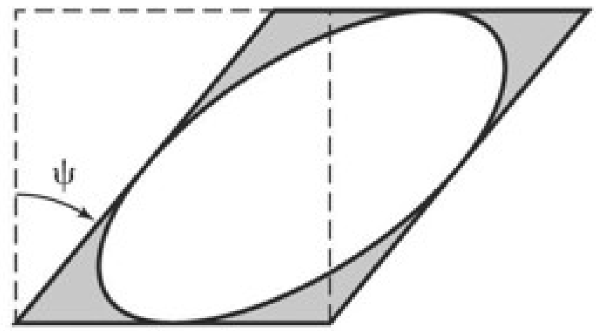
e = elongation (e1, e2, e3)

s = stretch = (1+e); X, Y, Z



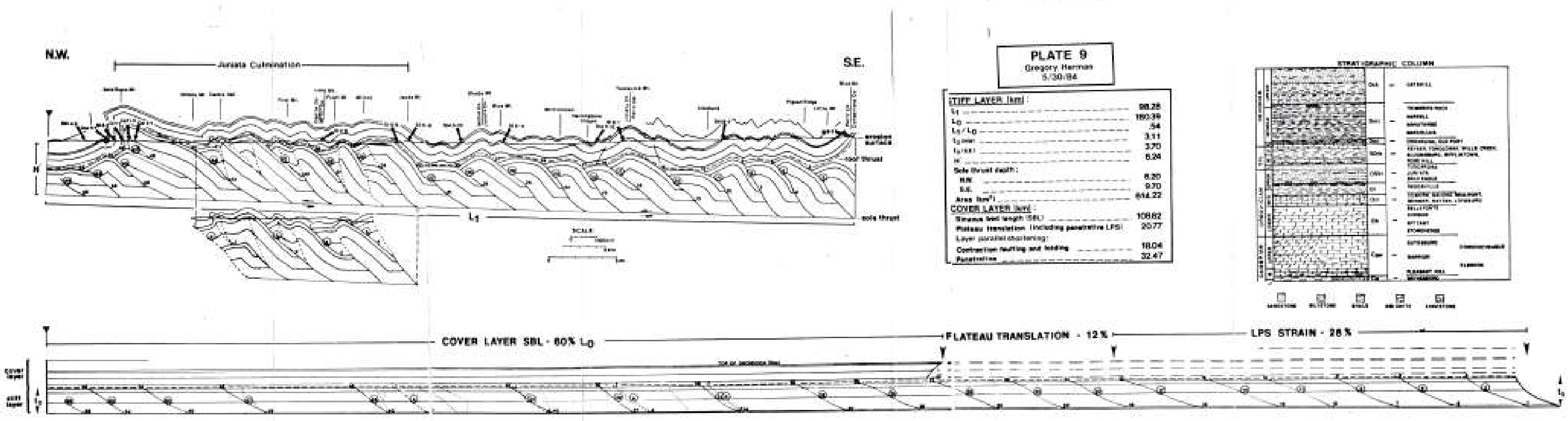
$$e = \frac{\ell - \ell_o}{\ell_o} = \frac{\delta\ell}{\ell_o} \;;\quad s = \frac{\ell}{\ell_o}$$

γ = shear strain (ψ is angular shear)



$\gamma = \tan\psi$

Herman, 1984 MSc Thesis page 121



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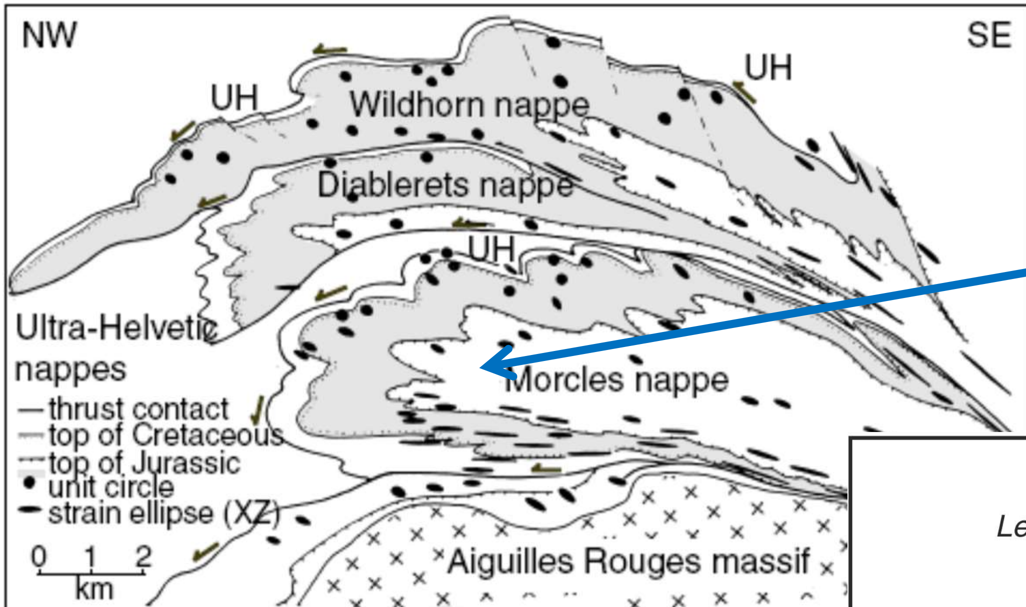
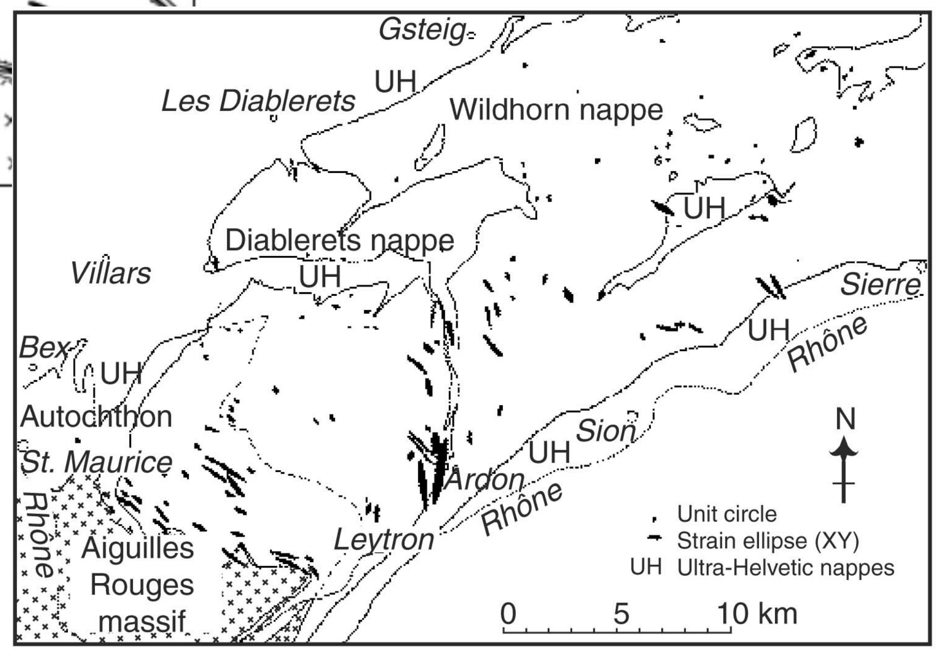
Representation of strain – map, section





Helvetic “nappes”, Switzerland.

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Strain

FIGURE 4 . 1 Deformed trilobites (Angelina sedgwicki) in a Cambrian slate from Wales. Knowledge about their original symmetry enables us to quantify the strain.

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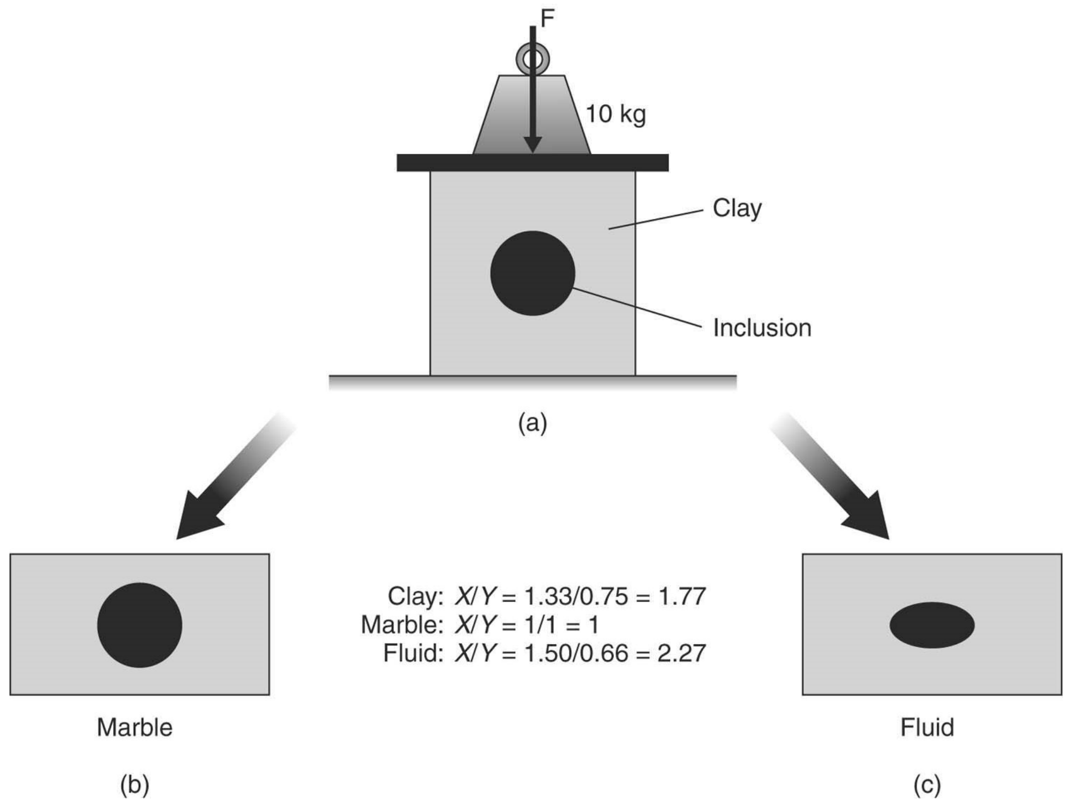
Strain and mechanical contrast

FIGURE 4 . 1 8 In two simple experiments we deform a cube of clay with a marble inclusion and a fluid inclusion.

If we require that the elongations in the clay are the same for both runs, the resulting elongations for the marble and fluid bubble are quite different.

This illustrates the different response to stress of materials with mechanical contrast or heterogeneous systems.

A practical example is the determination of strain using (strong) conglomerate clasts in a (weak) clay matrix.



Passive markers have no mechanical contrast: bulk rock strain

Active markers have mechanical contrast: marker strain

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Rheology

Rheology is

Associated concepts:

- Stress
- Strain, strain rate
- Elasticity
- Viscosity
- Failure and friction
- Plasticity
- Brittle behavior
- Ductile behavior

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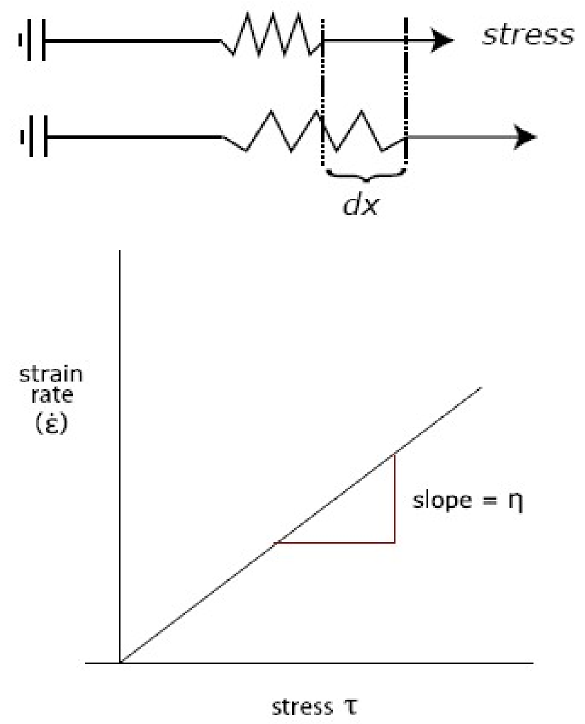
Definitions

- *A material's response to an imposed stress is called the rheology*.
- *A mathematical representation of the rheology is called a constitutive law.*

Ideal material behaviors

Simple, 1D Hooke body (linear elasticity)



Elastic behavior – Hooke's law

Viscous behavior – Newtonian fluids

Viscoelasticity - Kelvin and Maxwell bodies

Other ideal theologies

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Elastic Constants

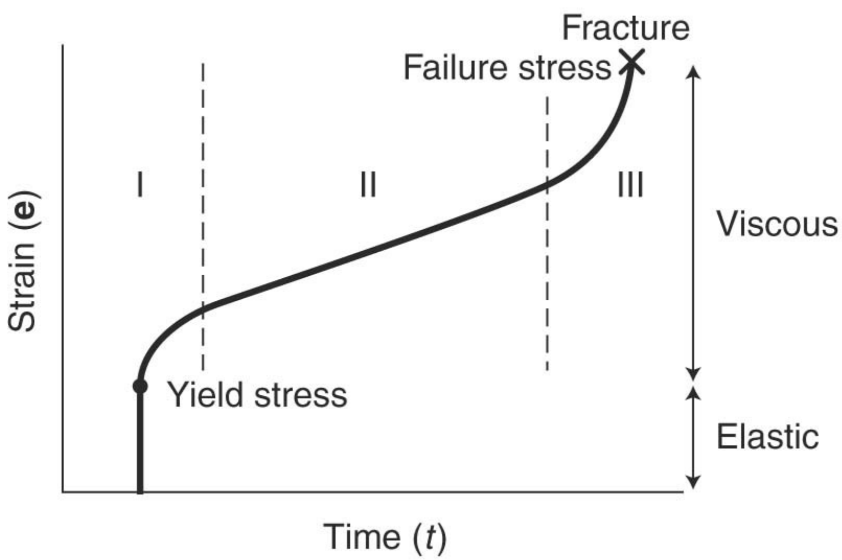
TABLE 5.4	ELASTIC CONSTANTS
Bulk modulus (*K*)	Ratio of pressure and volume change
Compressibility (1/*K*)	The inverse of the bulk modulus
Elasticity (*E*)	Young's modulus
Poisson's ratio (ν)	A measure of compressibility of a material. It is defined as the ratio between e normal to compressive stress and e parallel to compressive stress.
Rigidity (*G*)	Shear modulus
Shear modulus (*G*)	Ratio of the shear stress and the shear strain
Young's modulus (*E*)	Ratio of compressive stress and longitudinal strain

General Behavior: The Creep Curve

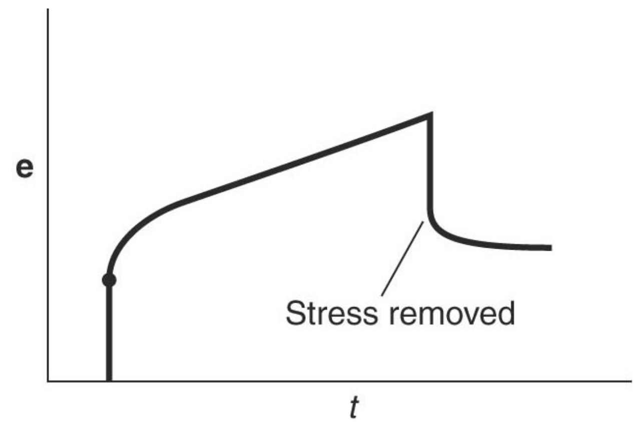
Generalized strain vs. time curve shows primary (I) or "transient", | secondary (II) or "steady-state", and tertiary (III) or "accelerated" creep.

Strain rate: time interval it takes to accumulate a certain amount of strain: $\dot{e} = e/t = \delta l/(l_o t)$

Shear strain rate: $\dot{\gamma} = 2\dot{e}$



Under continued stress the material will fail



When stress is removed, the material relaxes, but permanent strain remains

Time interval for 30% strain	$\dot{e}$
1 day $(86.4 \times 10^3$ s)	3.5×10^{-6}/s
1 year $(3.15 \times 10^7$ s)	9.5×10^{-9}/s
1 m.y. $(3.15 \times 10^{13}$ s)	9.5×10^{-15}/s

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Rheologic models

- Physical models consisting of strings and dash pots, and associated strain–time, stress–strain, or stress–strain rate curves for (a) *elastic*, (b) viscous, (c) viscoelastic, (d) elasticoviscous, and (e) *general linear behavior*.

- A useful way to examine these models is to draw your own strain–time curves by considering the behavior of the spring and the dash pot individually, and their interaction.

- An ideal **dashpot** or damper creates a force proportional to its velocity; it cannot be displaced instantaneously by a finite force.

Symbols used:

e = elongation, $\dot{\mathbf{e}}$ = strain rate, **σ**=stress, **E** = elasticity, **η** (eta)=viscosity, **t** = time, **el** denotes elastic component, **vi** denotes viscous component.

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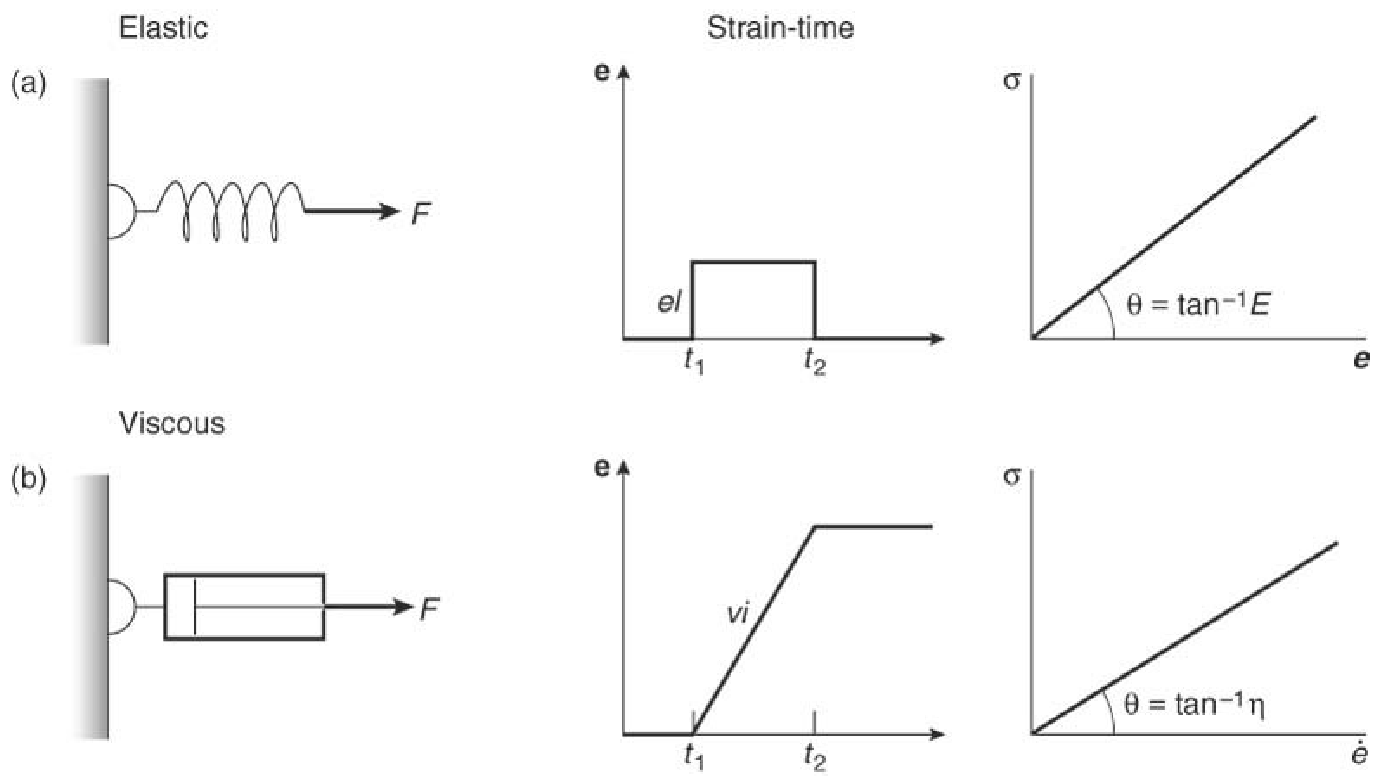
Rheologic models

FIGURE 5.3 Models of linear rheologies

a) *Elastic or Hookean behavior*: rubber band $\sigma = E \, . \, e \; (\sigma_s = G. \, \gamma)$



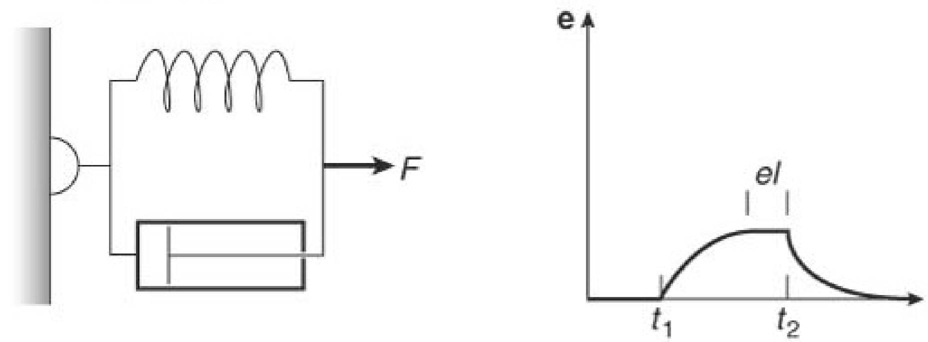
b) *Viscous or Newtonian behavior*: water

Symbols used: e = elongation, e˙ = strain rate, σ=stress, E = elasticity, η (eta)=viscosity, t = time, el denotes elastic component, vi denotes viscous component.

Rheologic models

Viscoelastic

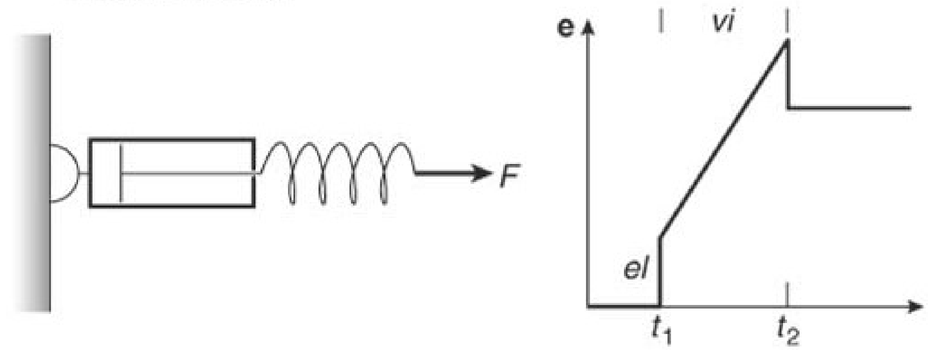
(c)



c) Viscoelastic or Kelvin behavior: water-soaked sponge, memory foam

Elastico-viscous

(d)

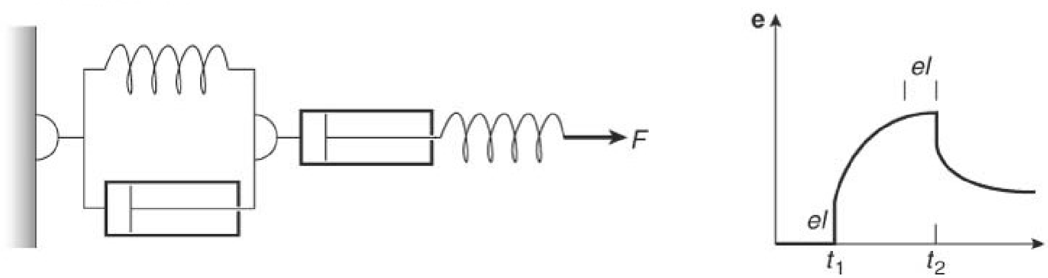


D) Elastoviscous or Maxwell behavior: paint

General linear

General linear

(e)



E) *General Linear behavior or Burgers behavior*

Symbols used: e = elongation, e˙ = strain rate, σ=stress, E = elasticity, η (eta)=viscosity, t = time, el denotes elastic component, vi denotes viscous component.

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Representative moduli and viscosities for rocks

TABLE 5.2	SOME REPRESENTATIVE BULK MODULI (*K*) AND SHEAR MODULI (OR RIGIDITY, *G*) IN 10^{11} Pa AT ATMOSPHERIC PRESSURE AND ROOM TEMPERATURE	
Crystal	*K*	*G*
Iron (Fe)	1.7	0.8
Copper (Cu)	1.33	0.5
Silicon (Si)	0.98	0.7
Halite (NaCl)	0.14	0.26
Calcite ($CaCO_3$)	0.69	0.37
Quartz (SiO_2)	0.3	0.47
Olivine (Mg_2SiO_4)	1.29	0.81
Ice (H_2O)	0.073	0.025

From Poirier (1985).

TABLE 5.5	REPRESENTATIVE VISCOSITIES (IN Pa · s)
Air	10^{-5}
Water	10^{-3}
Olive oil	10^{-1}
Honey	4
Glycerin	83
Lava	$10–10^{4}$
Asphalt	10^{5}
Pitch	10^{9}
Ice	10^{12}
Glass	10^{14}
Rock salt	10^{17}
Sandstone slab	10^{18}
Asthenosphere (upper mantle)	$10^{20}–10^{21}$
Lower mantle	$10^{21}–10^{22}$

From several sources, including Turcotte and Schubert (1982).

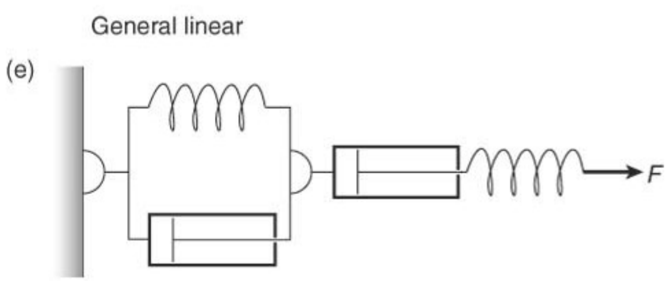
$K = \sigma/\Delta V \quad G = \sigma_s/\gamma \quad \eta = \sigma/e^o$

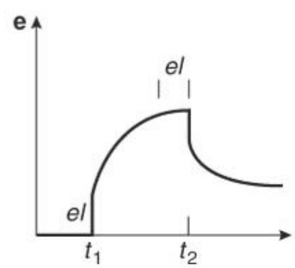
Non-linear model

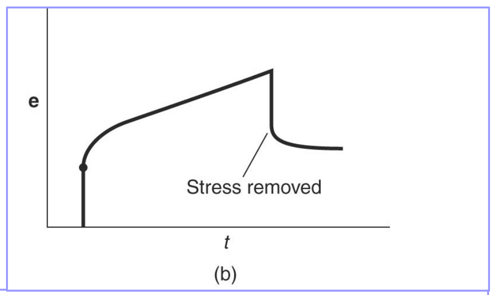
General linear behavior is modeled by placing the elastico-viscous and visco-elastic models in series.

Elastic strain accumulates at the first application of stress and subsequent behavior displays the interaction between the elastico-viscous and visco-elastic models.

When the stress is removed, the elastic strain is first recovered, followed by the viscoelastic component. However, some amount of strain (permanent strain) will remain, even after long time intervals (the viscous component of the elastico-viscous model).





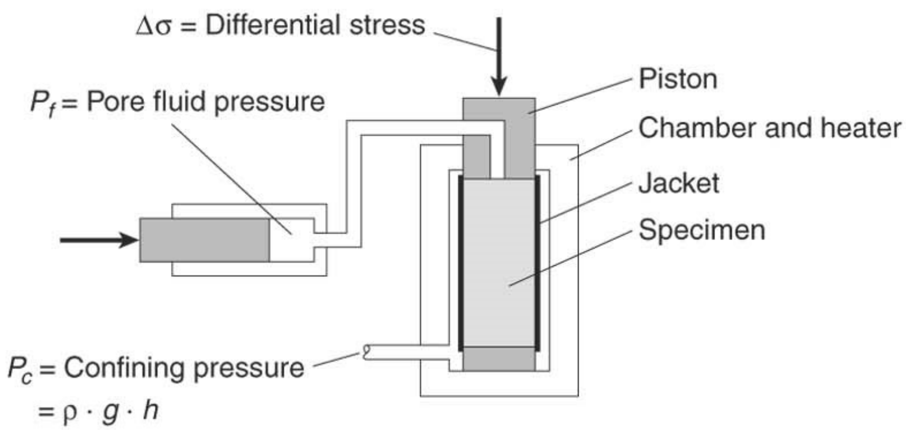


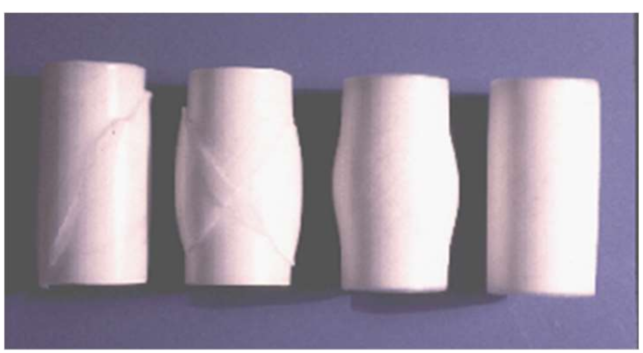
The creep curve for general linear closely mimics the creep curve that is observed in experiments on natural rocks

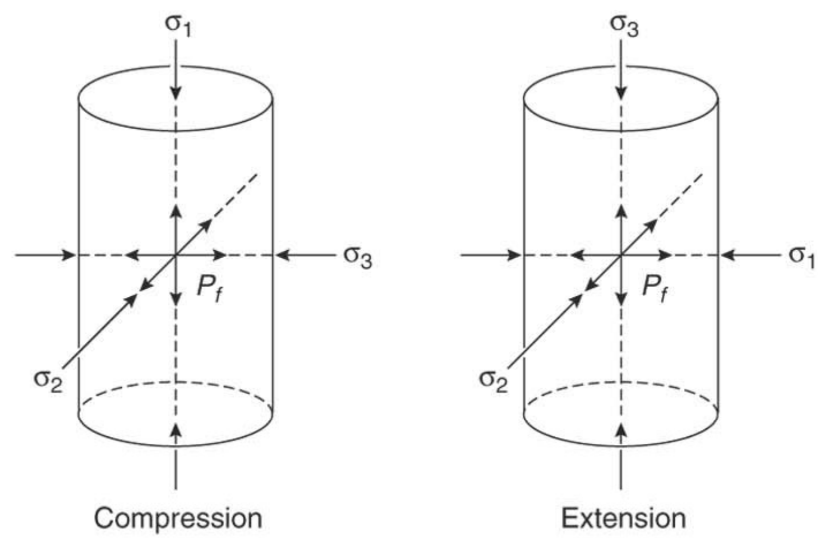
Rock experiments

FIGURE 5.8 Schematic diagram of a triaxial compression apparatus and states of stress in cylindrical specimens in compression and extension tests.

- The values of Pc, Pf, and σ can be varied during the experiments.







Compression:

$\sigma_1^* > \sigma_2^* = \sigma_3^* = P_c - P_f$

$\sigma_1^* = \Delta\sigma + P_c - P_f$

Extension:

$\sigma_3^* < \sigma_1^* = \sigma_2^* = P_c - P_f$

$\sigma_3^* = P_c - P_f - \Delta\sigma$

$\sigma_1^*, \sigma_2^*, \sigma_3^*$ = Maximum, intermediate, minimum effective principal stresses

$P_c - P_f$ = Effective pressure

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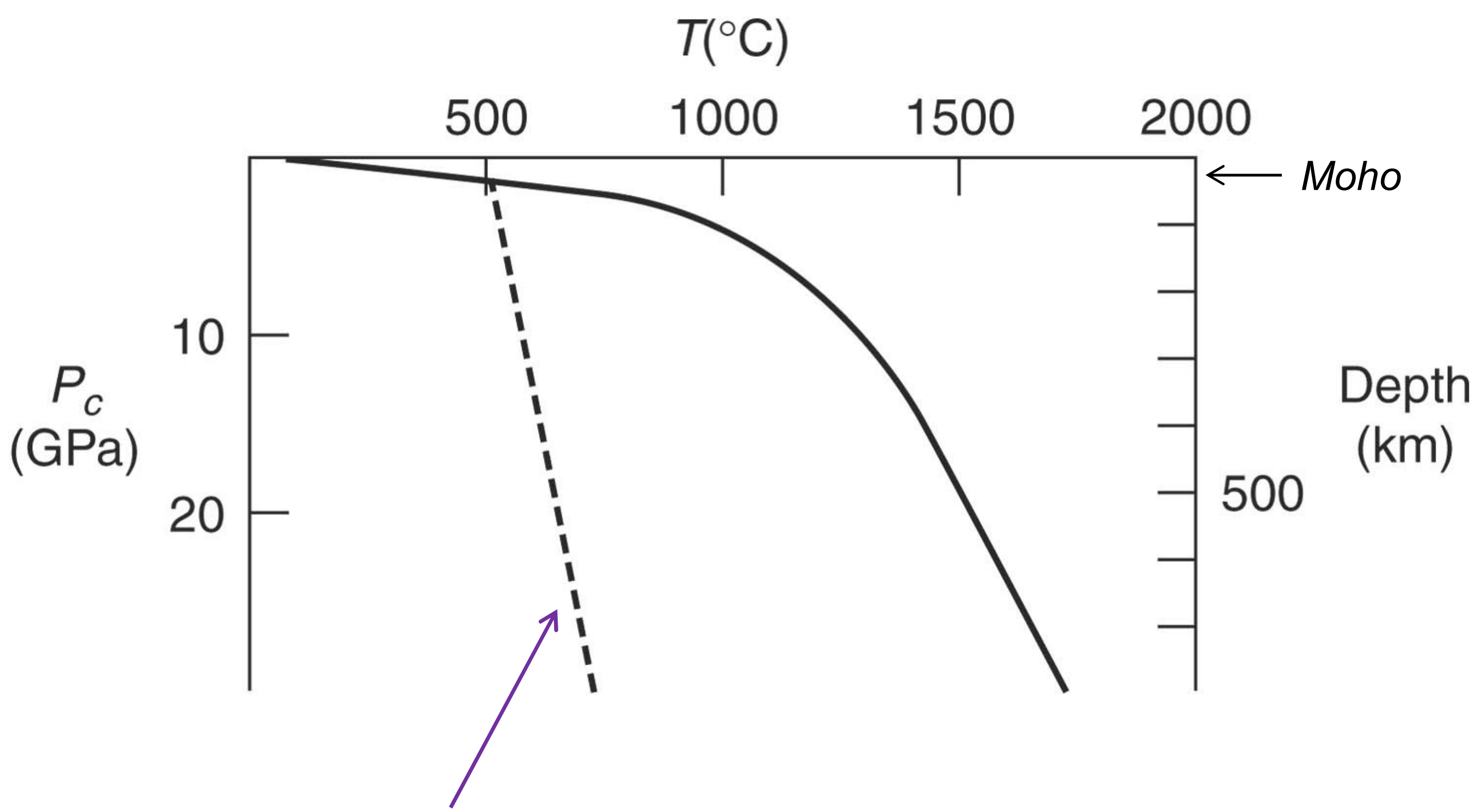
P, T, depth relationship

FIGURE 5.9 Change of temperature (T) and pressure (Pc) with depth.



The dashed line is the adiabatic gradient, which is the increase of temperature with depth resulting from increasing pressure and the compressibility of silicates.

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Pressure ***Suppresses fracturing, promotes ductility, increases strength***

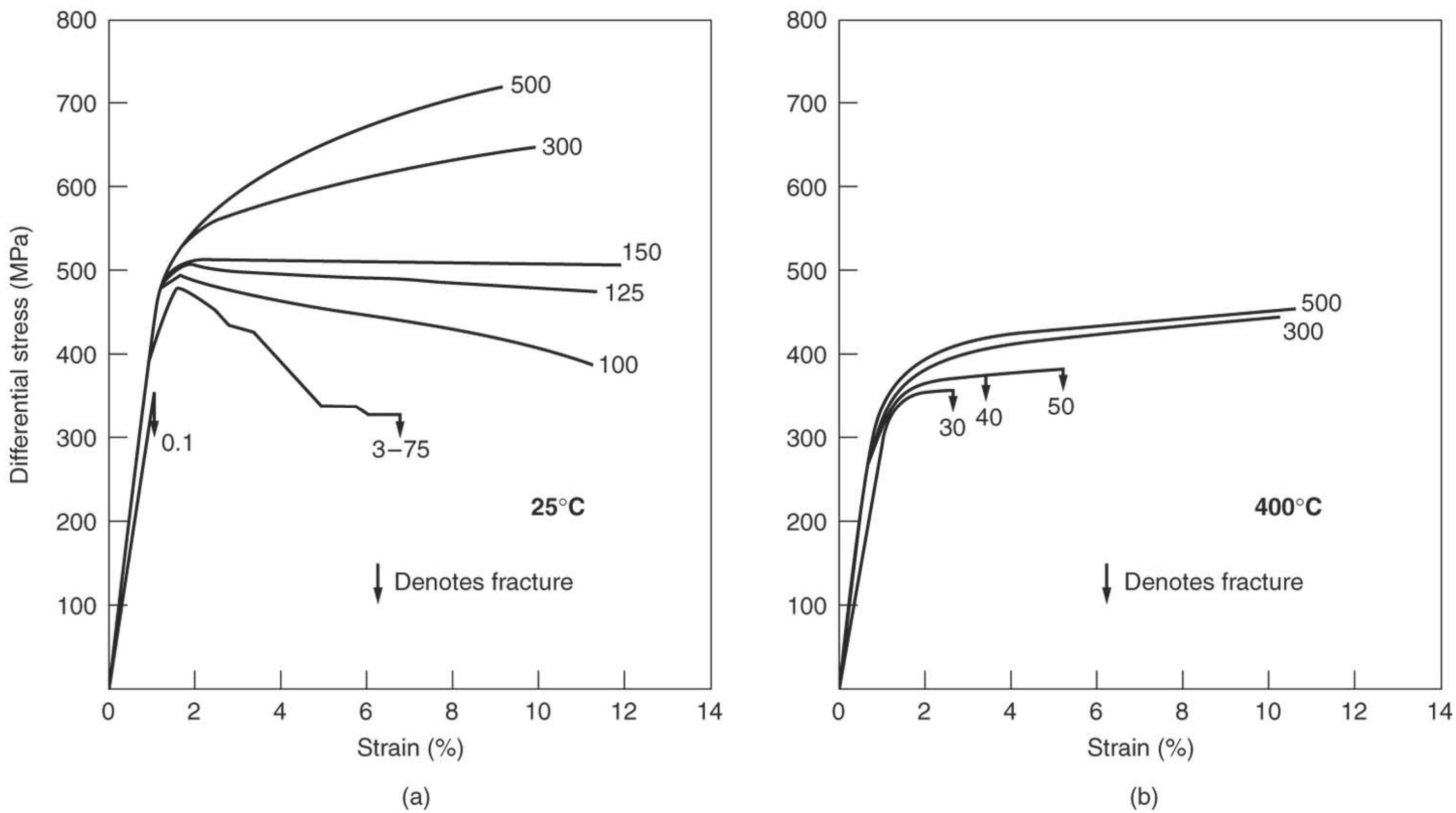

FIGURE 5.10 Compression stress–strain curves of Solnhofen limestone at various confining pressures (indicated in MPa) at (a) 25°C and (b) 400°C.

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Temperature ***Suppresses fracturing, promotes ductility, reduces strength***

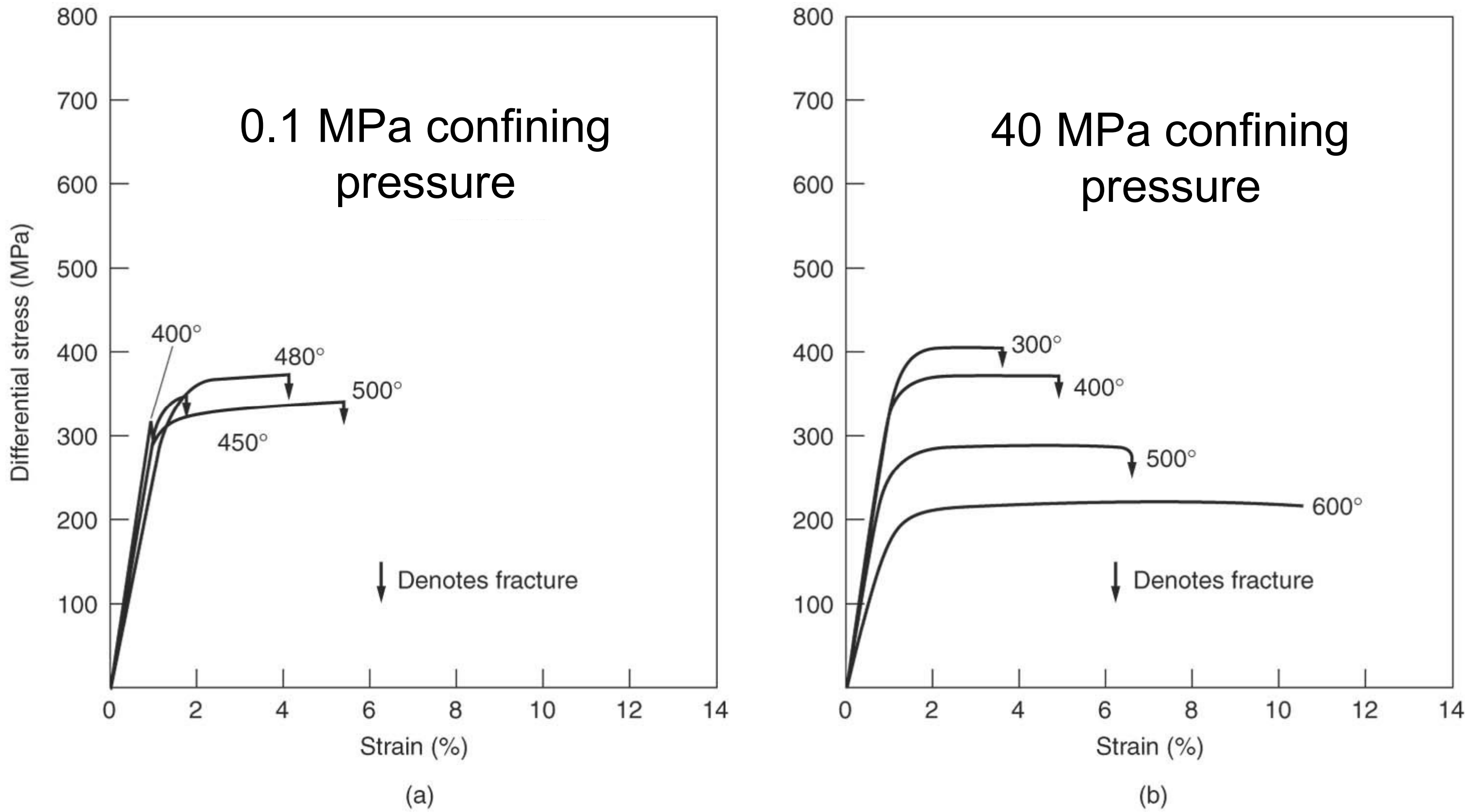

FIGURE 5.12 Compression stress–strain curves of Solnhofen limestone at various temperatures (indicated in °C) and pressures

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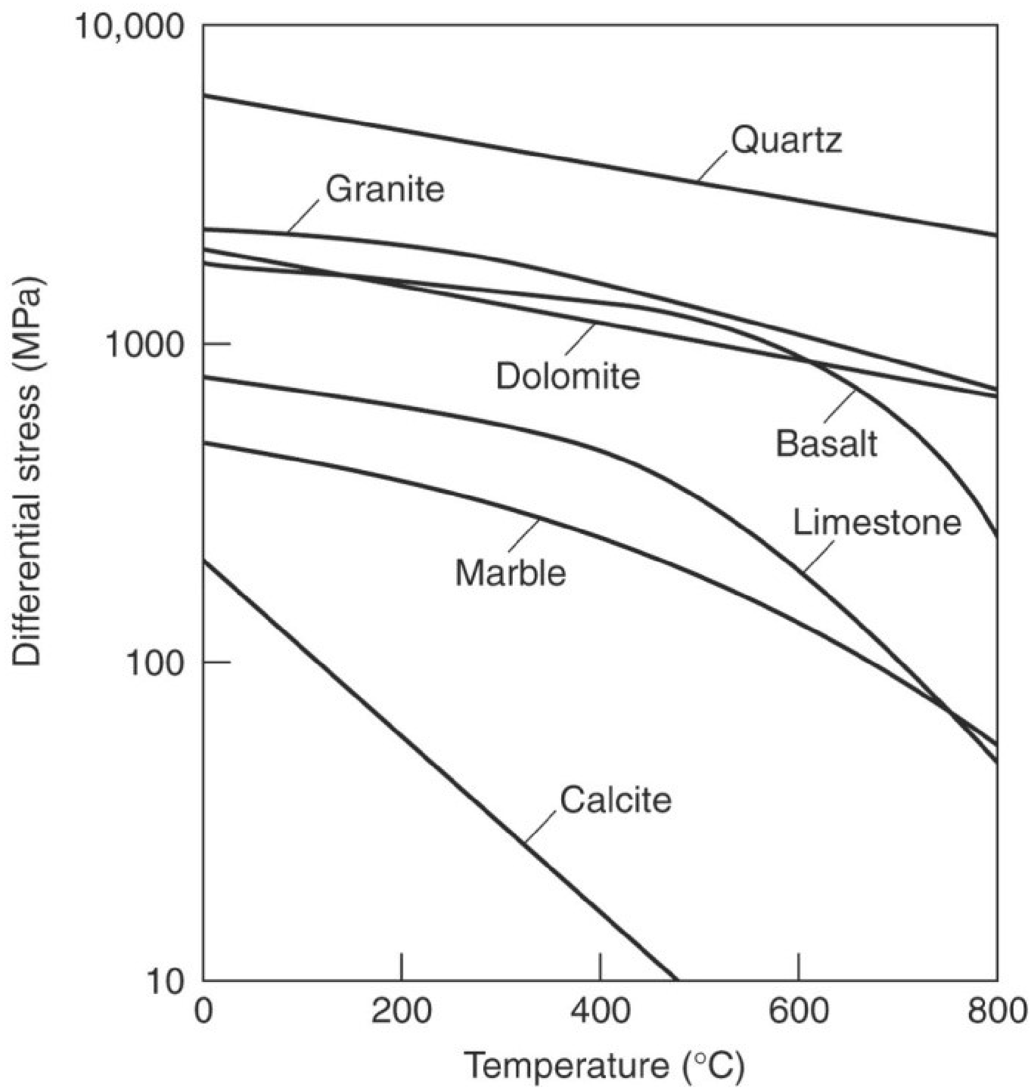
Temperature

FIGURE 5.13 The effect of temperature changes on the compressive strength of some rocks and minerals.

The maximum stress that a rock can support until it flows (called the **yield strength** of a material) decreases with increasing temperature.



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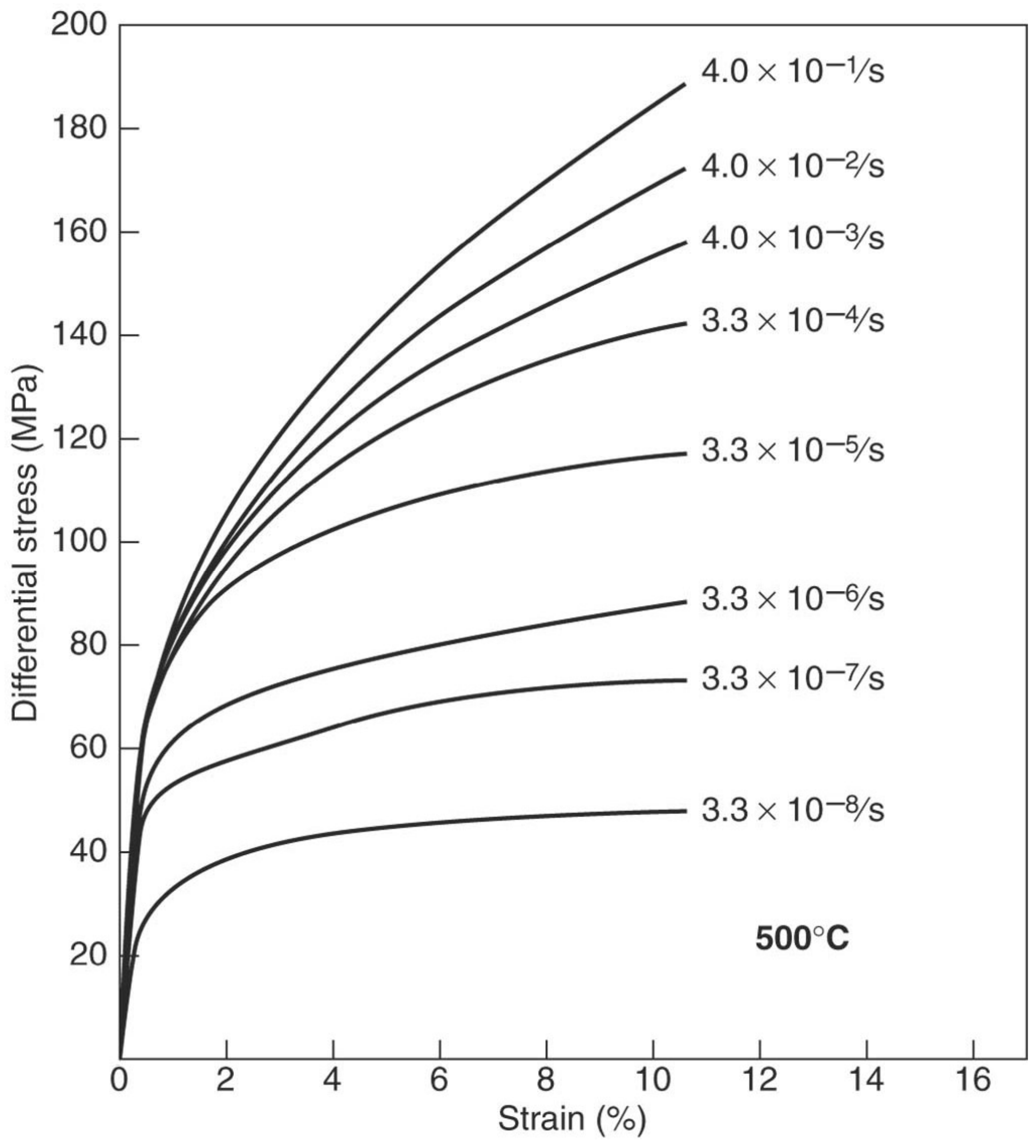
Strain rate

FIGURE 5.14 Stress versus strain curves for extension experiments in weakly foliated Yule marble for various constant strain rates at 500°C.

$e^o = 10^{-6}$/sec is 30% change in 4 days

$e^o = 10^{-14}$/sec is 30% change in 1 million years



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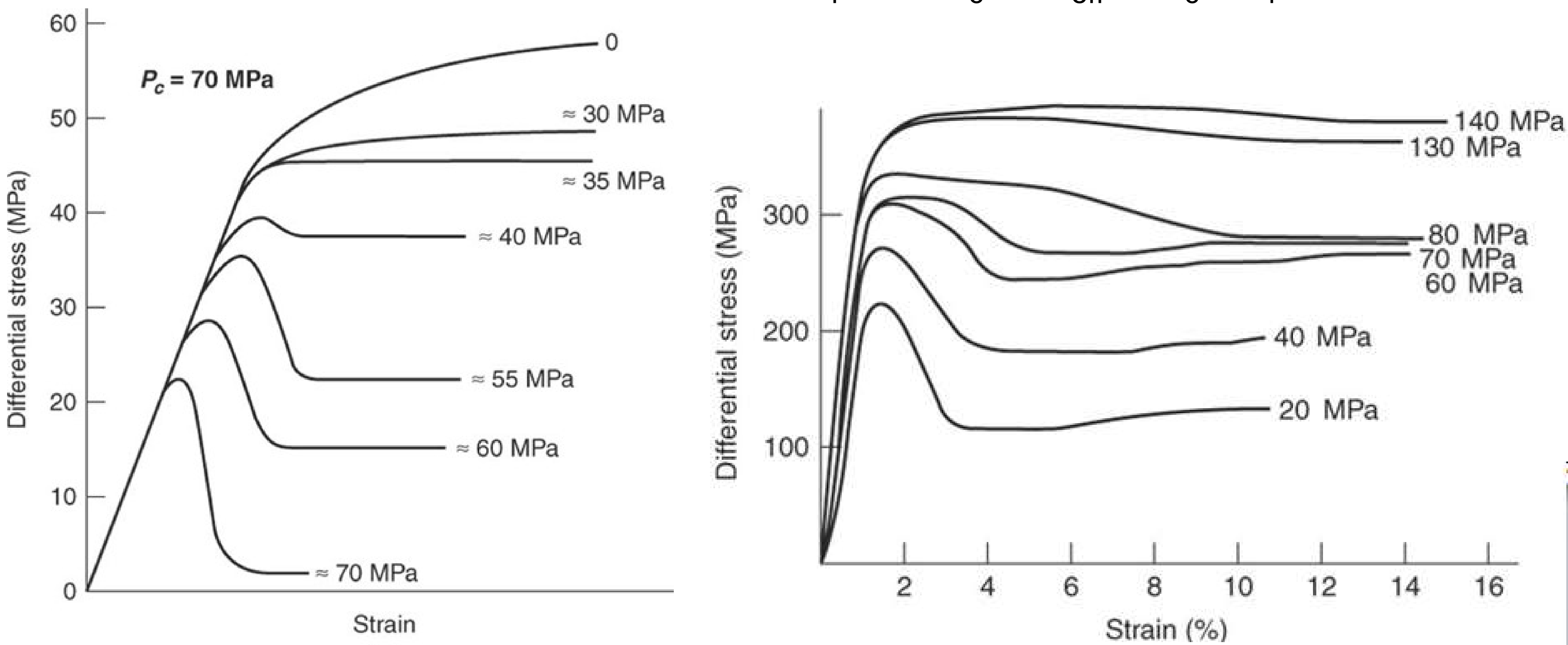
Fluid pressure

FIGURE 5.16 Comparing the effect of fluid pressure on the behavior of limestone

$P_f \sim 1/P_c \quad P_{eff} = P_c - P_f$



(a) varying pore-fluid pressure (b) **varying** confining pressure.

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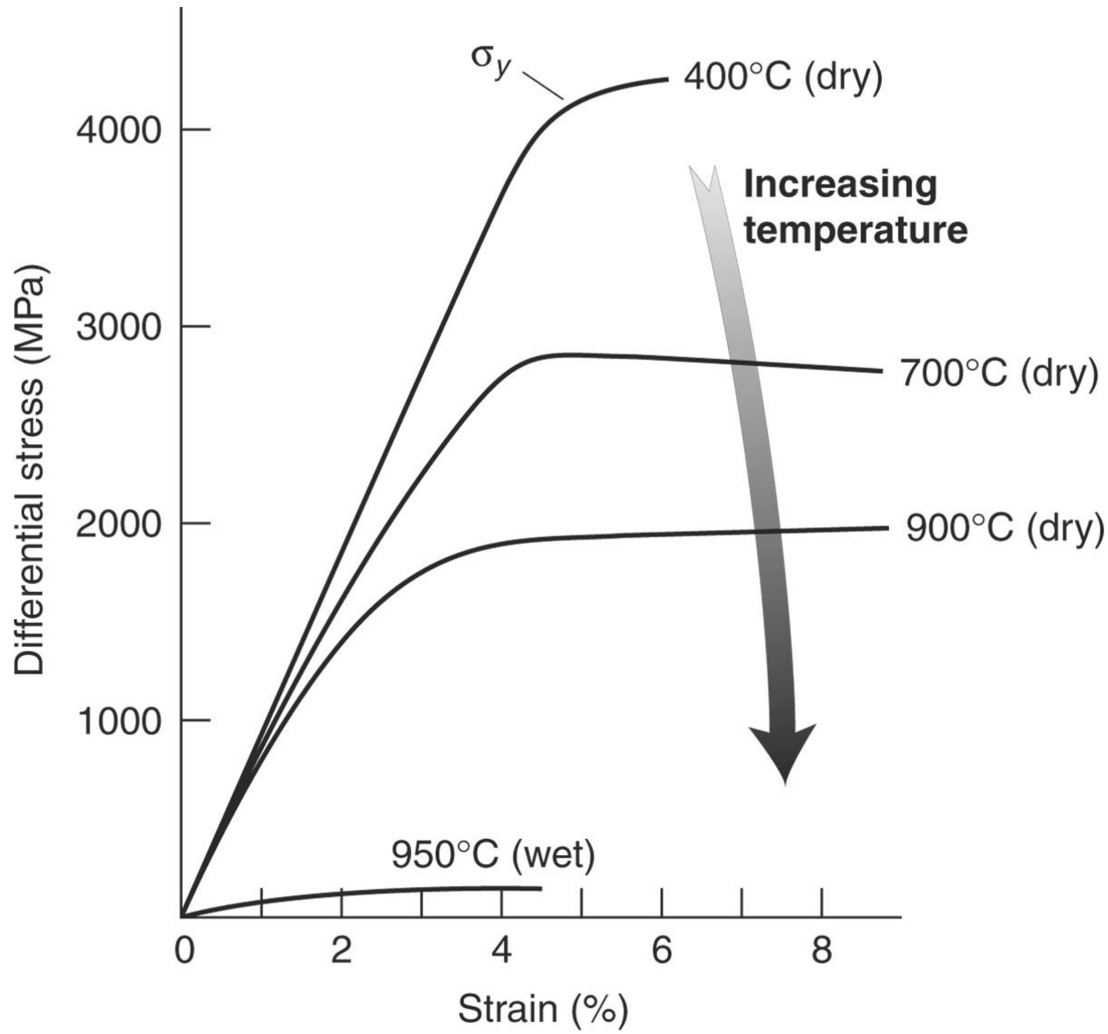
Pore-Fluid Pressure

- Natural rocks commonly contain a fluid phase that may originate from the depositional history or may be secondary in origin (for example, fluids released from prograde metamorphic reactions).
- In particular, low-grade sedimentary rocks such as sandstone and shale, contain a significant fluid component that will affect their behavior under stress.
- Experiments show that increasing the pore-fluid pressure produces a drop in the sample's strength and reduces the ductility.
- In other words, rocks are weaker when the pore-fluid pressure is high.

Fluid pressure

FIGURE 5.17 The effect of water content on the behavior of natural quartz.

Dry and wet refer to low and high water content, respectively.



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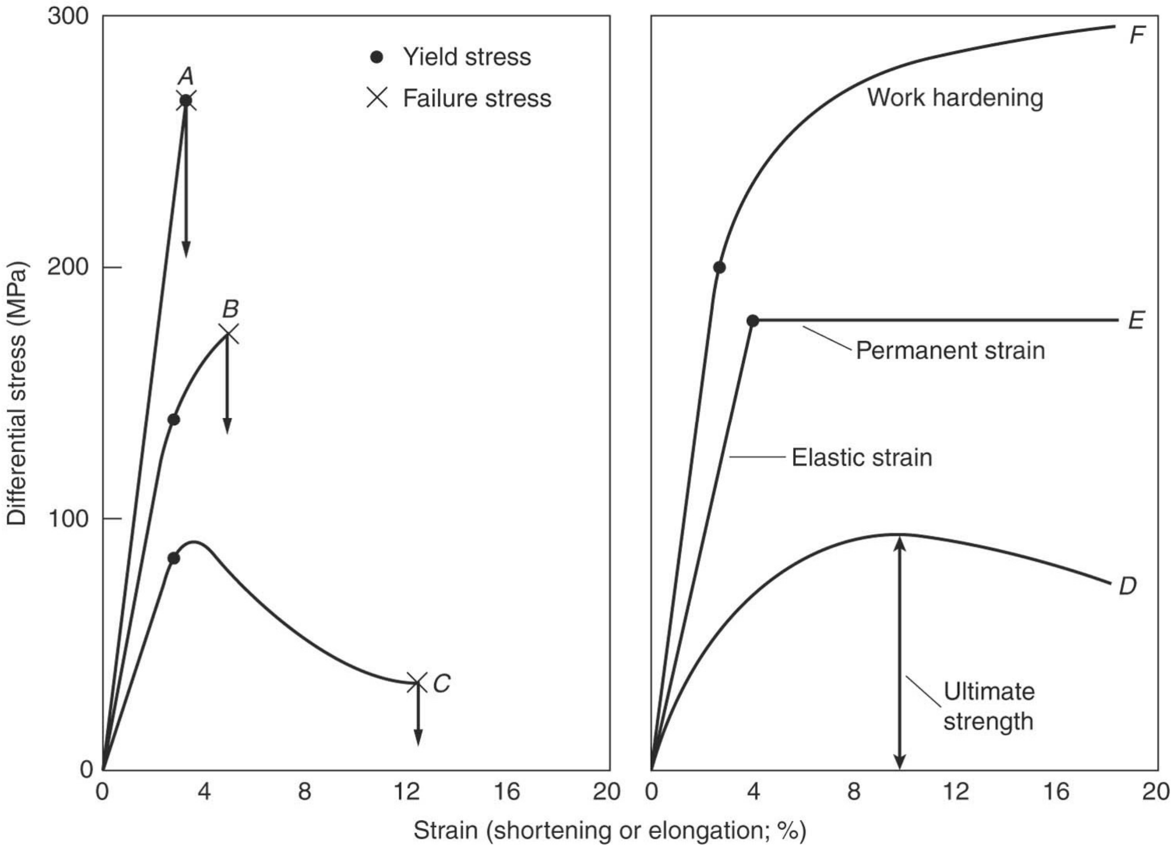
SIGNIFICANCE FOR GEOLOGIC CONDITIONS

TABLE 5.7	EFFECT OF ENVIRONMENTAL PARAMETERS ON RHEOLOGIC BEHAVIOR	
	Effect	**Explanation**
High P_c	Suppresses fracturing; increases ductility; increases strength; increases work hardening	Prohibits fracturing and frictional sliding; higher stress necessary for fracturing exceeds that for ductile flow
High T	Decreases elastic component; suppresses fracturing; increases ductility; reduces strength; decreases work hardening	Promotes crystal plastic processes
Low $\dot{e}$	Decreases elastic component; increases ductility; reduces strength; decreases work hardening	Promotes crystal plastic processes
High P_f	Decreases elastic component; promotes fracturing; reduces ductility; reduces strength *or* promotes flow	Decreases P_c ($P_e = P_c - P_f$) and weakens Si-O atomic bonds

Low Pc, high Pf, low T, high e°: promotes fracturing

High Pc, low Pf, high T, low e°: promotes flow

Representative stress-strain curves of brittle and ductile behavior



Yield stress marks stress at change from elastic (recoverable or nonpermanent strain) to viscous (nonrecoverable or permanent strain) behavior

Failure stress is stress at fracturing.

A elastic behavior followed immediately by failure, representing brittle behavior.

B, small viscous component (permanent strain) present before brittle failure.

C, permanent strain accumulates before material fails, representing transitional behavior between brittle and ductile.

D no elastic component and work softening.

E ideal elastic-plastic behavior, in which permanent strain accumulates at constant stress above yield stress.

F typical behavior of many experiments, displaying component of elastic strain followed by permanent strain that requires increasingly higher stresses to accumulate (*work hardening*).

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TABLE 5.8	TERMINOLOGY RELATED TO RHEOLOGY, WITH EMPHASIS ON BEHAVIOR AND MECHANISMS
Brittle-ductile transition	Depth in the Earth below which brittle behavior is replaced by ductile processes (see under "behavior" below)
Brittle-plastic transition	Depth in the Earth where the dominant deformation mechanism changes from fracturing to crystal plastic processes (see under "mechanisms" below)
Competency	Relative term comparing the resistance of rocks to flow
Failure stress	Stress at which failure occurs
Fracturing	Deformation mechanism by which a rock body or mineral loses coherency
Crystal plasticity	Deformation mechanism that involves breaking of atomic bonds without the material losing coherency
Strength	Stress that a material can support before failure
Ultimate strength	Maximum stress that a material undergoing work softening can support before failure
Work hardening	Condition in which stress necessary to continue deformation experiment increases
Work softening	Condition in which stress necessary to continue deformation experiment decreases
Yield stress	Stress at which permanent strain occurs
Material behavior	
Brittle behavior	Response of a solid material to stress during which the rock loses continuity (cohesion). Brittle behavior reflects the occurrence of brittle deformation mechanisms. It occurs only when stresses exceed a critical value, and thus only occurs after the body has already undergone some elastic and/or plastic behavior. The stress necessary to induce brittle behavior is affected strongly by pressure (stress-sensitive behavior); brittle behavior generally does not occur at high temperatures.
Ductile behavior	A general term for the response of a solid material to stress such that the rock appears to flow mesoscopically like a viscous fluid. In a material that has deformed ductilely, strain is distributed, i.e., strain develops without the formation of mesoscopic discontinuities in the material. Ductile behavior can involve brittle (cataclastic flow) or plastic deformation mechanisms.

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Elastic behavior	Response of a solid material to stress such that the material develops an instantaneous, recoverable strain that is linearly proportional to the applied stress. Elastic behavior reflects the occurrence of elastic deformation mechanisms. Rocks can undergo less than a few percent elastic strain before they fail by brittle or plastic mechanisms, and conditions of failure are dependent on pressure and temperature during deformation.
Plastic behavior	Response of a solid material to stress such that when stresses exceed the yield strength of the material, it develops a strain without loss of continuity (i.e., without formation of fractures). Plastic behavior reflects the occurrence of plastic deformation mechanisms, is affected strongly by temperature, and requires time to accumulate (strain rate–sensitive behavior).
Viscous behavior	Response of a liquid material to a stress. As soon as the differential stress becomes greater than zero, a viscous material begins to flow, and the flow rate is proportional to the magnitude of the stress. Viscous deformation takes time to develop.
Deformation mechanisms	
Brittle deformation mechanisms	Mechanisms by which brittle deformation occurs, namely fracture growth and frictional sliding. Fracture growth includes both joint formation and shear rupture formation, and sliding implies faulting. If fracture formation and frictional sliding occur at a grain scale, the resulting deformation is called cataclasis; if cataclasis results in the rock "flowing" like a viscous fluid, then the process is called cataclastic flow.
Elastic deformation mechanisms	Mechanisms by which elastic behavior occurs, namely the bending and stretching, without breaking, of chemical bonds holding atoms or molecules together.
Plastic deformation mechanisms	Mechanisms by which plastic deformation occurs, namely dislocation glide, dislocation creep (glide and climb; including recovery, recrystallization), diffusive mass transfer (grain-boundary diffusion or Coble creep, and diffusion through the grain or Herring-Nabarro creep), grain–boundary sliding/superplasticity.

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Brittle and Ductile Behavior

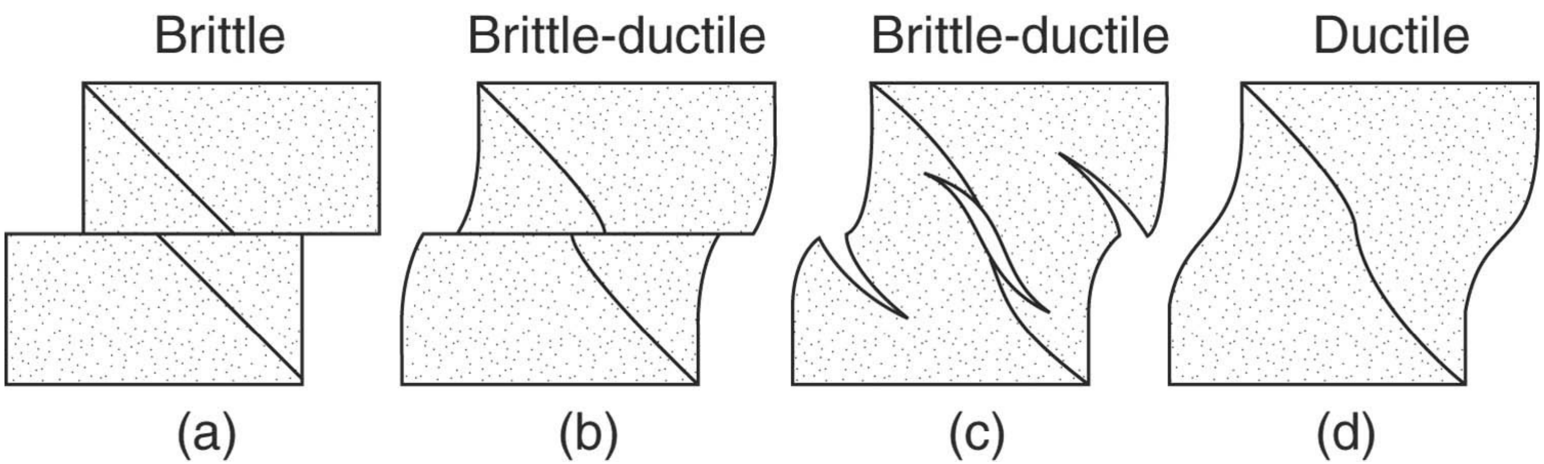

FIGURE 5.21 Brittle (a) to brittle-ductile (b, c) to ductile (d) deformation,

Brittle behavior: response of a solid material to stress during which rock loses continuity (cohesion). Brittle behavior reflects the occurrence of fracture mechanisms. It occurs only when stresses exceed a critical value, after body has already undergone some elastic and/or plastic behavior.

Ductile behavior: response of a solid material to stress such that the rock appears to flow mesoscopically like a viscous fluid. In a material that has deformed ductilely, strain is distributed, i.e., strain develops without formation of mesoscopic discontinuities. Ductile behavior can involve brittle (cataclastic flow) or plastic deformation mechanisms.

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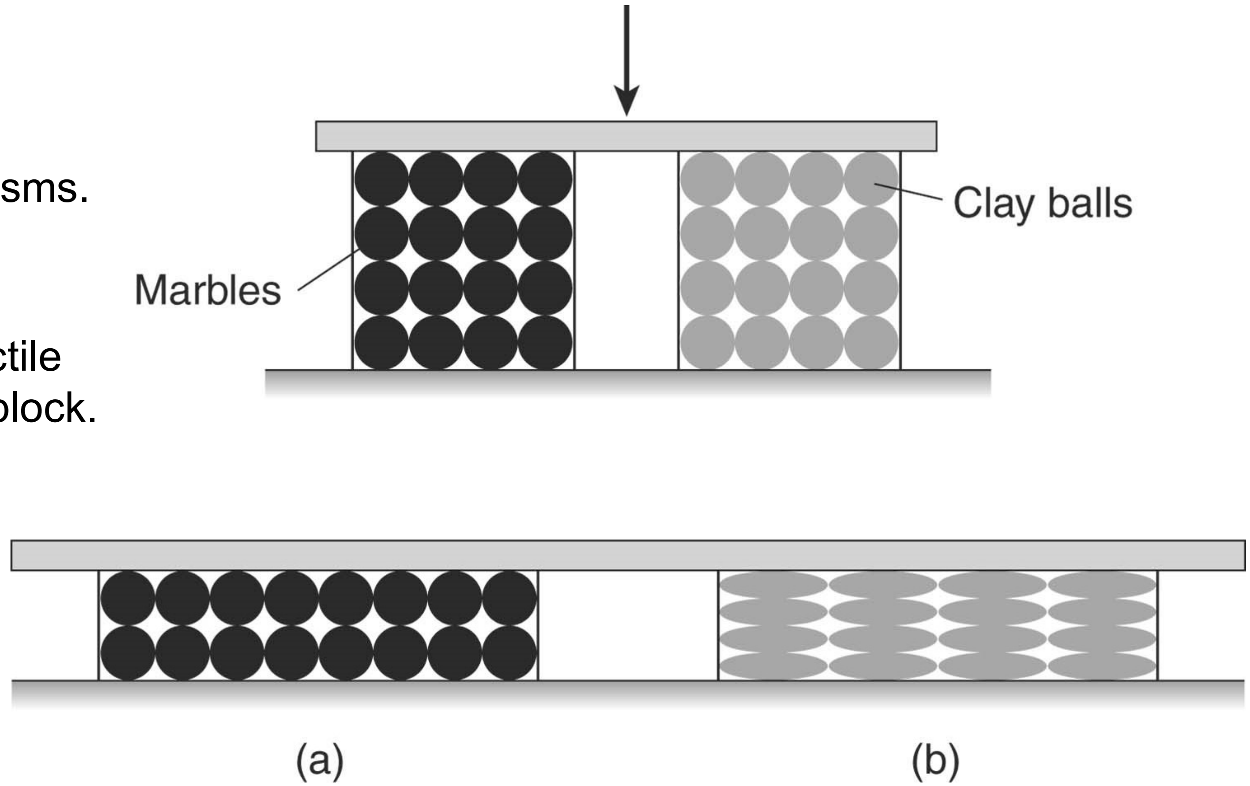
Ductile strain: behavior vs. mechanism

FIGURE 5.19 Deformation experiment with two cubes containing marbles (a), and balls of clay (b)

Ductile strain accumulates by different deformation mechanisms.

The finite strain is equal in both cases and the mode of deformation is distributed (ductile behavior) on the scale of the block.

However, the mechanism by which the deformation occurs is quite different



frictional sliding of undeformed marbles

Plastic distortion of clay balls into ellipsoids.

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Strength and Competency

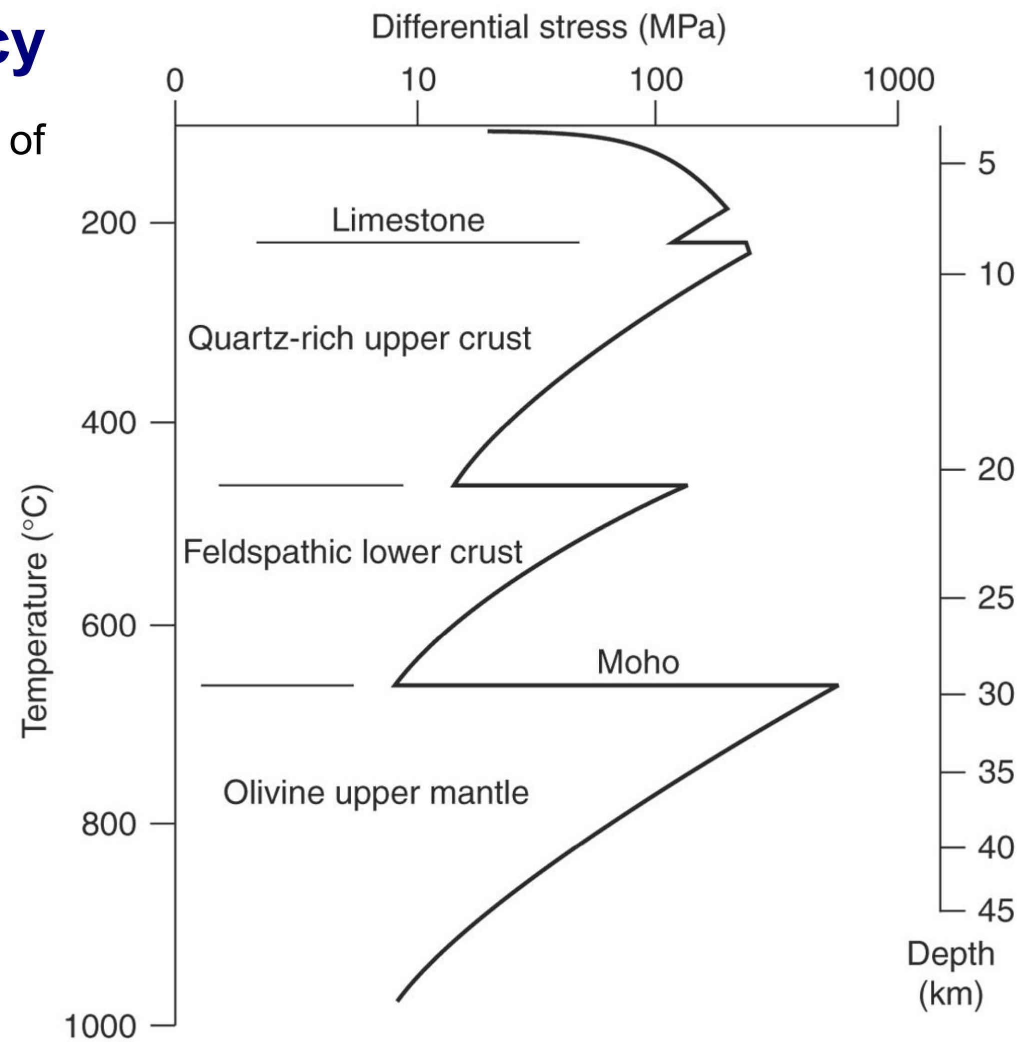
FIGURE 5.20 ***Rheological stratification*** of the lithosphere based on the mechanical properties of characteristic minerals.

Computed lithospheric strength (i.e., the differential stress) changes not only as a function of composition, but also as a function of depth (i.e., temperature).

Strength is stress that material can support before failure.

Competency is *relative* term that compares resistance of rocks to flow.

Rock competency scale:
rock salt < shale < limestone
< greywacke < sandstone < dolomite
< schist < marble < quartzite < gneiss < granite < basalt



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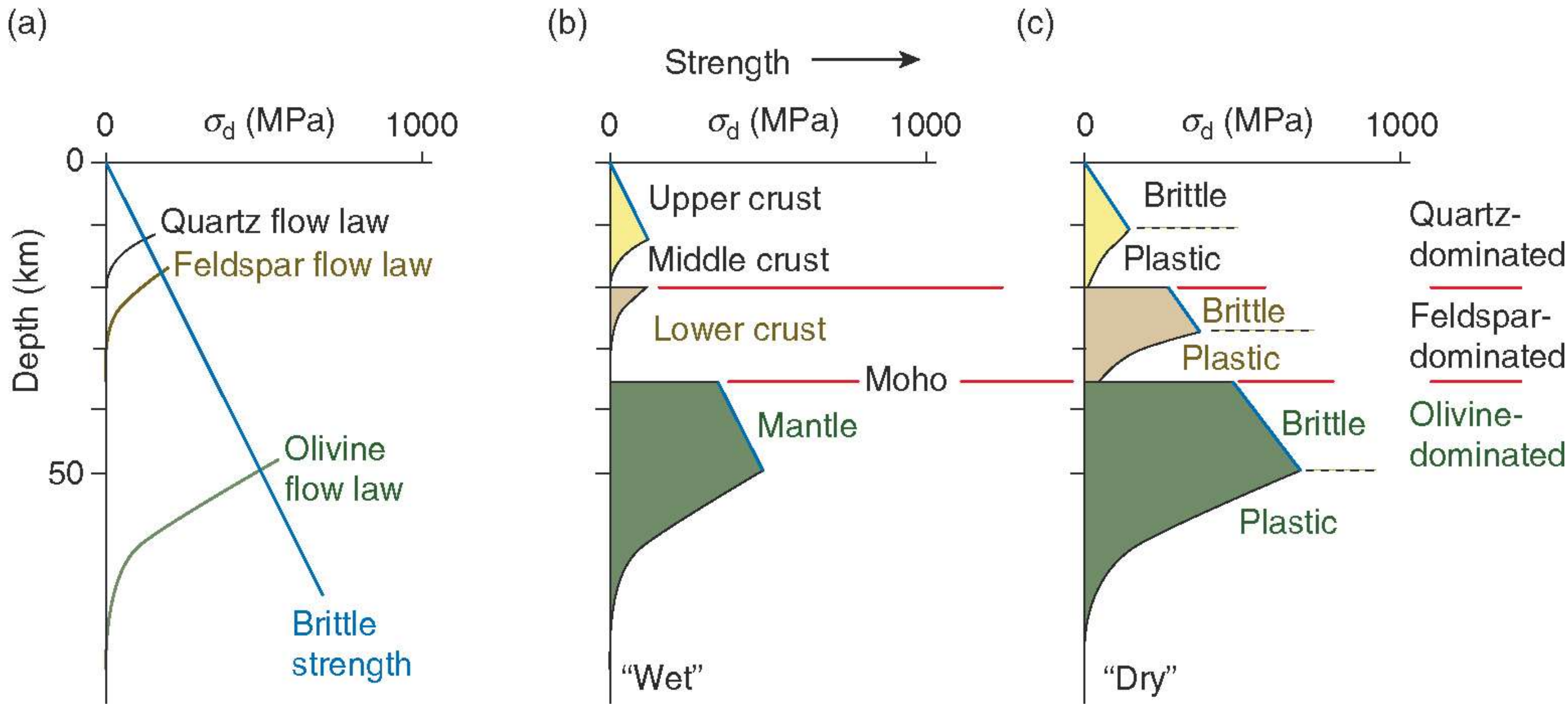
Strength and Rheologic Stratification



Combining friction and flow laws (dry and wet) (Fossen 2010)

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