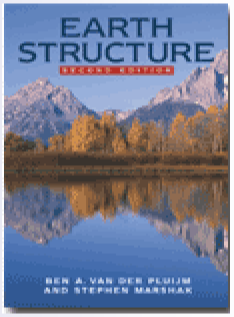
Lecture 3

CONTINUUM MECHANICS - MATERIAL STRESS CONCEPTS



Earth Structure (2nd Edition), 2004

W.W. Norton & Co, New York

Slide show by Ben van der Pluijm

CONTINUUM MECHANICS - MATERIAL STRESS CONCEPTS

- In tectonic structures we commonly deal with interactions that involve both movement and distortion; material displacements occur within and between bodies.

- Observations are constrained within a Cartesian coordinate reference frame in order to gauge magnitude and bearing

- *Continuum mechanics treat material as continuous medium*

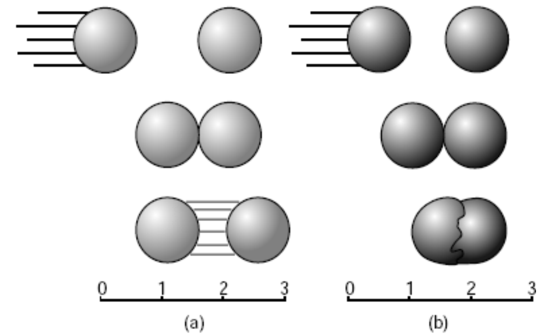


FIGURE 3.2 The interaction of nondeformable bodies is described by Newtonian (or classical) mechanics (a) and that between deformable bodies by continuum mechanics (b). Imagine the difference between playing pool with regular balls and balls made of jelly.

- **Force = mass * acceleration** *(mass in kilograms and acceleration in m/s^2)*

- Forces that result from action of a field at every point within the body are called **body forces.** (Example gravity acting on objects with mass)

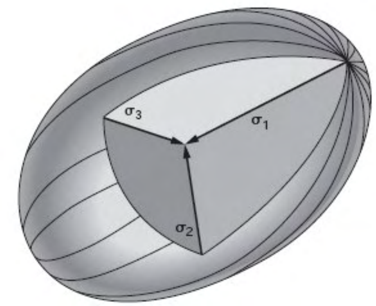
- Forces that act on a specific surface area in a body are called **surface forces.** They reflect the pull or push of the atoms on one side of a surface against the atoms on the other side. (Examples cue stick's force on a pool ball)

CONTINUUM MECHANICS - MATERIAL STRESS CONCEPTS

Body forces acting on point in 3D within a body is described by a ***stress ellipse and tensor***

In engineering, e.g., structural, mechanical, or geotechnical, the stress distribution within an object, for instance stresses in a rock mass around a tunnel, airplane wings, or building columns, is determined through a stress analysis. Calculating the stress distribution implies the determination of stresses at every point (material particle) in the object. According to Cauchy, the *stress at any point* in an object (Figure 2), assumed as a continuum, is completely defined by the nine stress components σ_{ij} of a second order tensor of type (2,0) known as the Cauchy stress tensor, $\boldsymbol{\sigma}$: **(sigma)**

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \equiv \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \equiv \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$



- The stress tensor includes three principal stresses, or three mutually perpendicular axes of the stress ellipsoid.
- These axes are perpendicular to three principal planes having no shear stresses.
- The state of stress in a body can be simply specified using the orientations and magnitude of *three principal stresses of the stress ellipsoid*.

CONTINUUM MECHANICS - MATERIAL STRESS CONCEPTS

- The stress state of a body is either *isotropic or anisotropic*, the latter being more ordinary

- **Isotropic** is when the three principal stresses are equal in magnitude (stress sphere rather than an ellipsoid because all three radii are equal) .

- **Anisotropic** when any of the principal stresses are unequal in magnitude

- *By geologic convention:* σ_1 = maximum principal stress
  σ_2 = intermediate principal stress
  σ_3 = minimum principal stress

- And thus: $\sigma_1 \geq \sigma 2 \geq \sigma_3$

Uniaxial test with three strain gauges attached

Several common stress states:

Hydrostatic stress (pressure): $\sigma_1 = \sigma 2 = \sigma_3 \neq 0$

General triaxial stress: $\sigma_1 > \sigma 2 > \sigma_3 \neq 0$

Biaxial (plane) stress: one axis = 0 (ex. $\sigma 1 > 0 > \sigma_3$)

Uniaxial compression: $\sigma_1 > 0; \sigma 2 = \sigma_3 = 0$

Uniaxial tension: $\sigma_1 = \sigma 2 = 0; \sigma_3 < 0$

CONTINUUM MECHANICS - MATERIAL STRESS CONCEPTS

- **Force applied to a surface produces surface stress**
- **Surface stress –** (Force per unit area) varies in intensity with the size of the plane
- The surface stress acting on a 2D plane has both *magnitude and direction* that produces ***traction***
- Traction is resolved into *normal (*σ_n) *and shear (*$\sigma_{s\ or}$ τ (tau)) *stress components* acting perpendicular and along the plane, respectively
- *Thus, the stress vector acting on a plane can be resolved into perpendicular and parallel vector components to that plane*

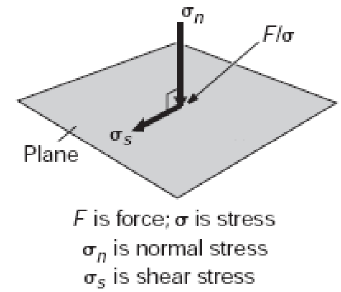


FIGURE 3.3 The stress on a two-dimensional plane is defined by a stress acting perpendicular to the plane (the normal stress) and a stress acting along the plane (the shear stress). The normal stress and shear stress are perpendicular to one another.

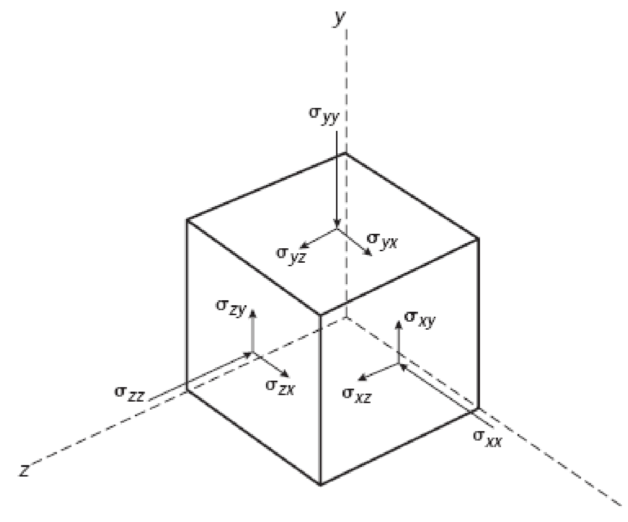


FIGURE 3.6 Resolution of stress into components perpendicular (three normal stresses, σ_n) and components parallel (six shear stresses, σ_s) to the three faces of an infinitesimally small cube, relative to the reference system *x*, *y*, and *z*.

CONTINUUM MECHANICS - MATERIAL STRESS CONCEPTS

- The trigonometric equations for the stress components for a plane depend upon the angle θ (theta) relative to the *minimum* principal stress direction:

$\sigma_n = 1/2(\sigma_1 + \sigma_3) + 1/2(\sigma_1 - \sigma_3)\cos 2\theta$ (eq. 3.7)

$\sigma_s = 1/2(\sigma_1 - \sigma_3)\sin 2\theta$ (eq. 3.10)

- *A force directed along, or parallel with a plane has zero (0) resolved normal stress and maximum shear stress*
- *A plane oriented normal to a force has the maximum resolved normal stress and zero shear stress*
- *A plane angled at 45º to a directed force has the **maximum shear stress** and an intermediate level of normal stress*

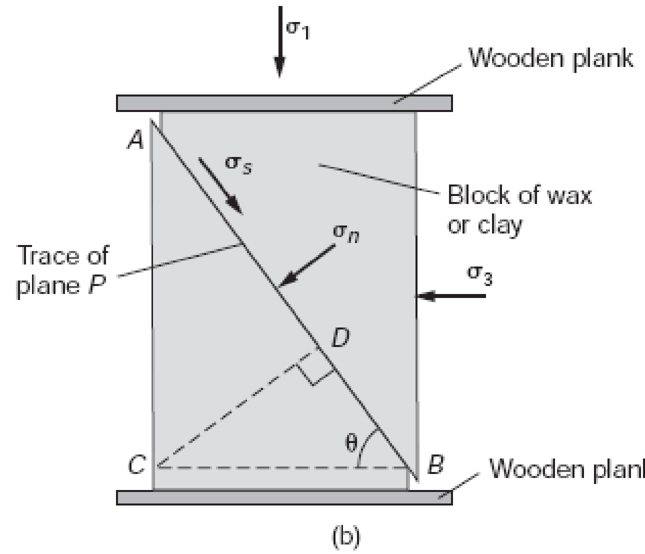


FIGURE 3.7 Determining the normal and shear stresses on a plane in a stressed body as a function of the principal stresses. (a) An illustration from the late nineteenth-century fracture experiments of Daubrée using wax. (b) For a classroom experiment, a block of clay is squeezed between two planks of wood. *AB* is the trace of fracture plane *P* in our body that makes an angle θ with σ_3. The two-dimensional case shown is sufficient to describe the experiment, because σ_2 equals σ_3 (atmospheric pressure).

Uniaxial compression: $\sigma_1 > 0$; $\sigma_2 = \sigma_3$

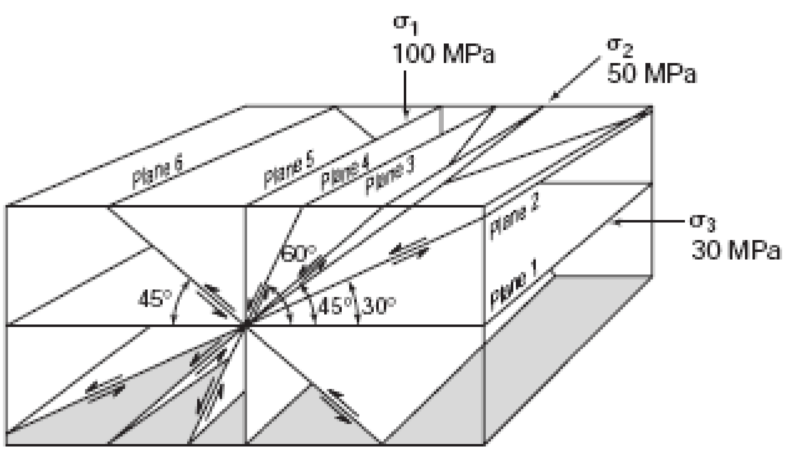
- # Mohr Diagrams

- Equations derived for σ_n and σ_s do not offer an obvious sense of their values as a function of orientation of a plane in a stressed body.

- Simple computer program are available but a graphical method known as the **Mohr diagram** (Figure 3.8), was introduced over a century ago to solve Equations 3.7 and 3.10.

- A Mohr diagram is an "*XY*"-type (Cartesian) plot of σ_s versus σ_n that graphically solves the equations for normal stress and shear stress acting on a plane within a stressed body.

- *In our experiences, many people find the Mohr construction difficult to comprehend.*



(a)

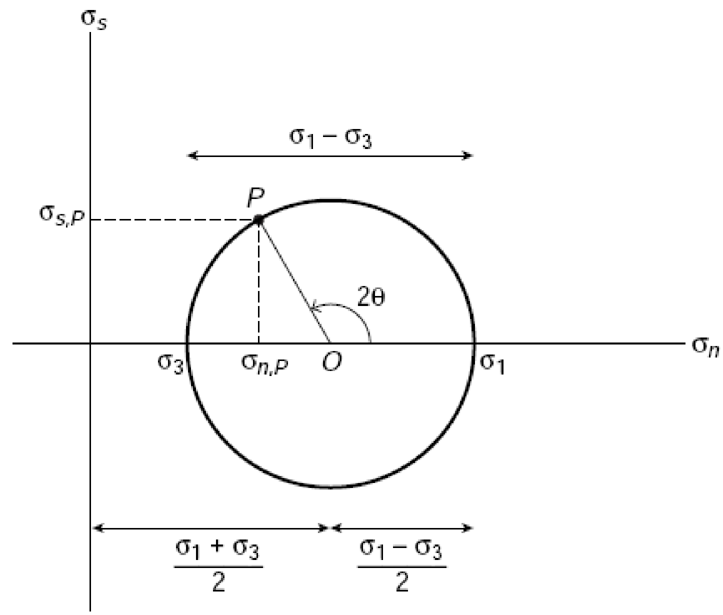
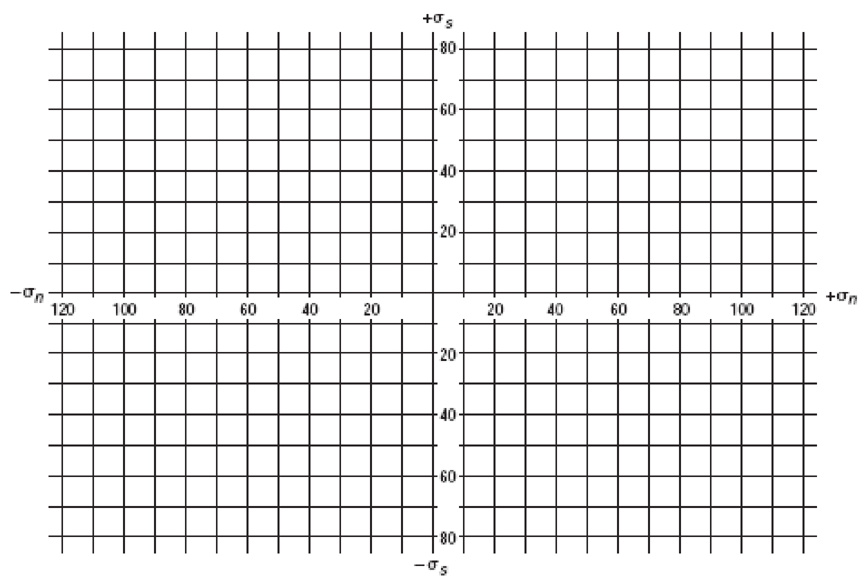


FIGURE 3.8 The Mohr diagram for stress. Point *P* represents the plane in our clay experiment of Figure 3.7.



EXPERIMENTAL DERIVATION OF STRESS STATES DURING MATERIAL FAILURE

SCIENTIFIC REPORTS | ARTICLE OPEN
www.nature.com/srep/2013/130123/srep01117/full/srep01117.html

A universal fracture criterion for high-strength materials

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Affiliations | Contributions | Corresponding author

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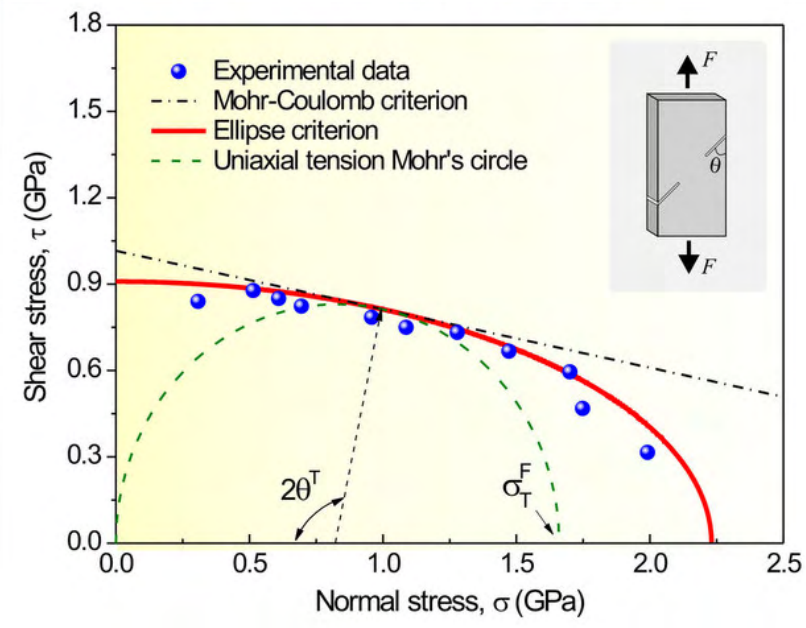
Recently developed advanced high-strength materials like metallic glasses, nanocrystalline metallic materials, and advanced ceramics usually fracture in a catastrophic brittle manner, which makes it quite essential to find a reasonable fracture criterion to predict their brittle failure behaviors. Based on the analysis of substantial experimental observations of fracture behaviors of metallic glasses and other high-strength materials, here we developed a new fracture criterion and proved it effective in predicting the critical fracture conditions under complex stress states. The new criterion is not only a unified one which unifies the three classical failure criteria, i.e., the maximum normal stress criterion, the Tresca criterion and the Mohr-Coulomb criterion, but also a universal criterion which has the ability to describe the fracture mechanisms of a variety of different high-strength materials under various external loading conditions.

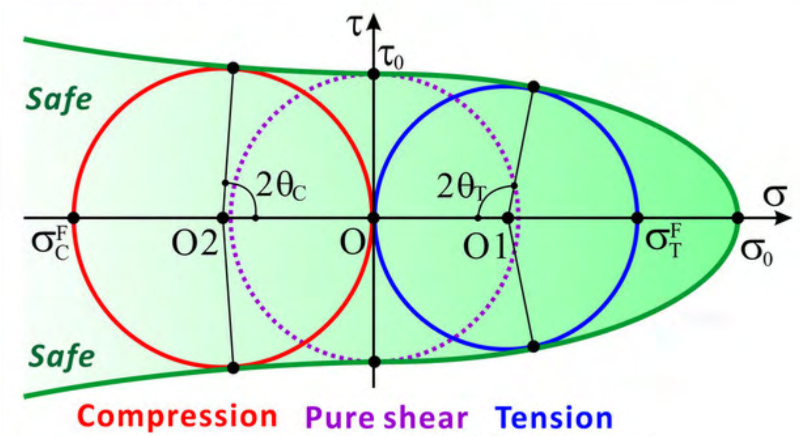
Subject terms: Metals and alloys • Glasses • Mechanical properties • Applied physics

Figure 1: Quantitatively experimental verification of the Ellipse criterion.



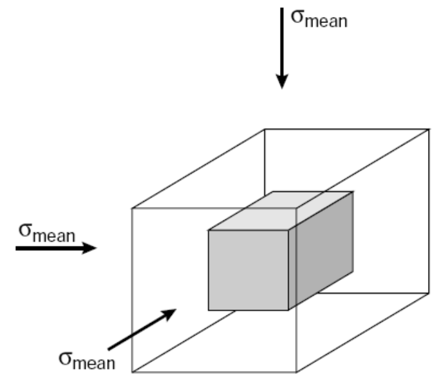
The experimental data indicates the Ellipse criterion can well predict the tensile fracture behaviors of the $Zr_{52.5}Cu_{17.9}Ni_{14.6}Al_{10}Ti_5$ MG[20]. Inset is the sketch of the designed sample.

Figure 2: Graphical representation of the new fracture criterion.

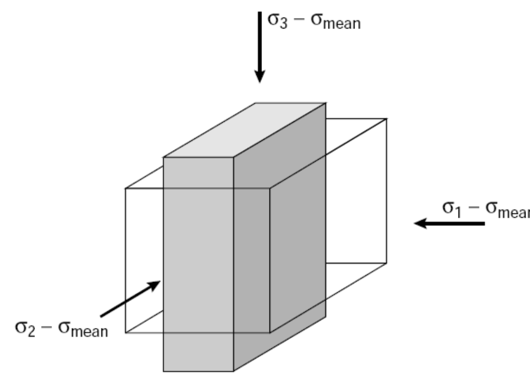


SOME COMMON AND USEFUL STRESS RELATIONSHIPS

- **Differential stress** ($\sigma_d = \sigma_1 - \sigma_3$) is the difference between maximum and minimum stresses
- *The differential stress is always twice the maximum shear stress* $1/2(\sigma_1 - \sigma3)$
- **Mean stress** ($\sigma_m = \sigma_1 + \sigma2 + \sigma_3$) / 3) is often called the *hydrostatic pressure because it exerted equally in all principal directions.*
- **Deviatoric stress** ($\sigma_{dev} = \sigma - \sigma_m$) is the difference between the total and mean stress.
- **In geology, we recognize *lithostatic pressure,* the component of in-situ stress resulting from lithostatic loading, or thickness above a depth point**



(a) *Mean stresses cause volume changes*



(b) *Deviatoric stress cause shape changes*

SOME COMMON AND USEFUL STRESS RELATIONSHIPS

- The *lithostatic stress component (or pressure)* is best explained by a simple but powerful calculation. The local pressure is a function of rock density, depth, and gravity:

Lithostatic pressure = $P_l = \rho \cdot g \cdot h$ (Eq. 3.16)

- Consider a rock at a depth of 3 km in the middle of a continent. The lithostatic pressure at this point is a function of the weight of the overlying rock column because other (tectonic) stresses are unimportant. If ρ (density) equals a representative crustal value of 2700 kg/m3, *g* (gravity) is 9.8 m/s^2, and *h* (depth) is 3000 m, we get:

P_l = 2700 · 9.8 · 3000 = 79.4 · 106 Pa ≈ 80 Mpa (or 800 bars)

- *For every kilometer in the Earth's crust the lithostatic pressure increases by approximately 27 MPa*.

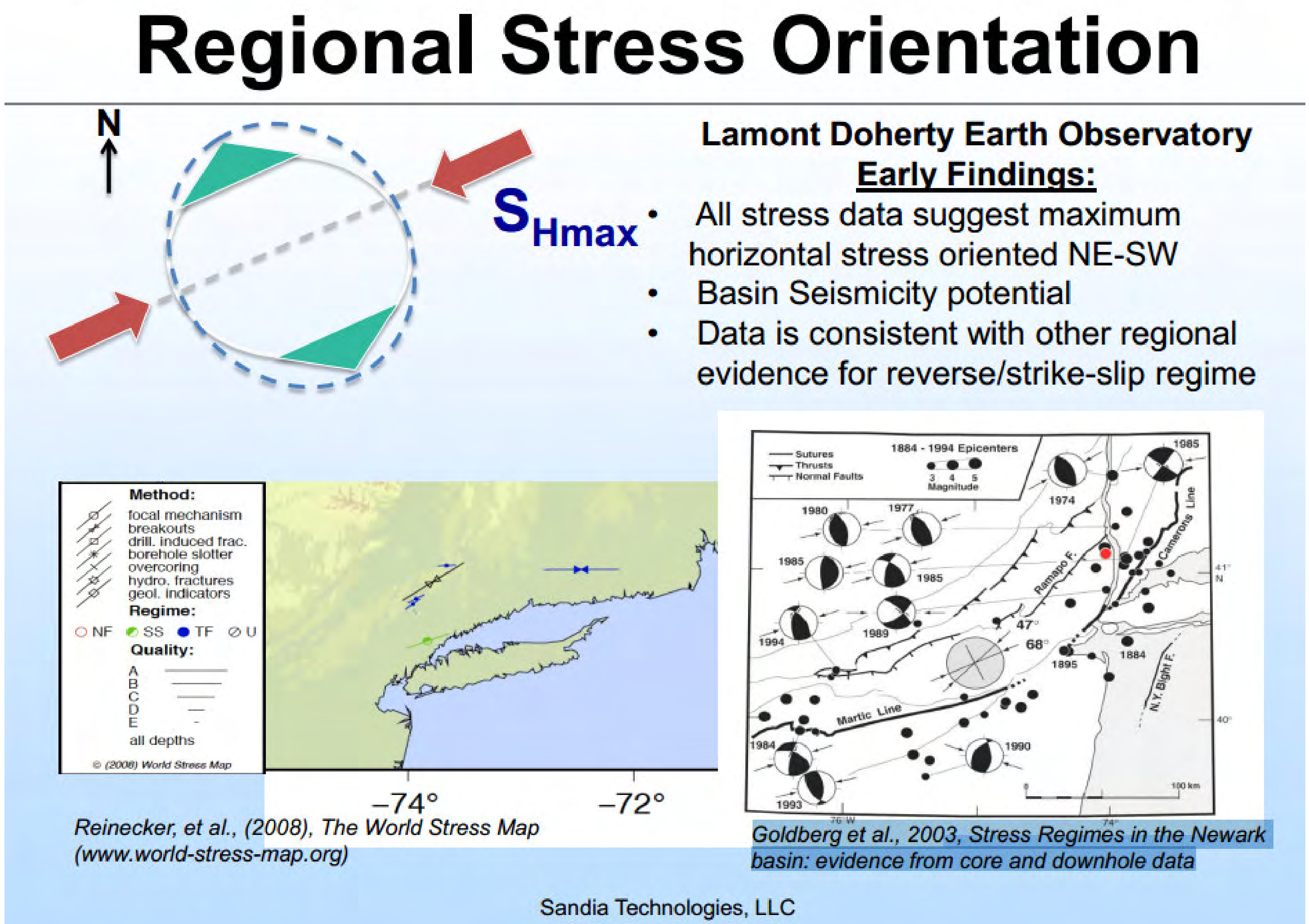
- But the density of rocks increases with depth: at about 15 km depth the average density of the crust is 2900 kg/m^3 , and reaches as much as 13,000 kg/m^3 in the solid inner core.

STRESS FIELD OF THE EARTH'S CRUST

is described using three, compressive principal components or axes

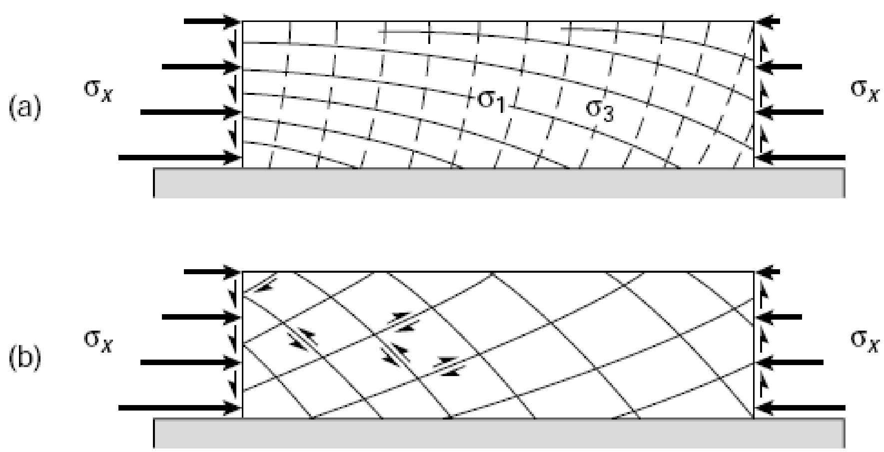
Two horizontal tectonic stresses: *minimum* (S_{HMIN}) and *maximum* (S_{HMAX}) and the *vertical* stress due to lithostatic pressure (S_V)

www.netl.doe.gov/File%20Library/Events/2012/Carbon%20Storage%20RD%20Project%20Review%20Meeting/Papadeas.pdf



STRESS TRAJECTORIES AND STRESS FIELDS

- **Stress trajectories –** Lines connecting the orientation of principal stress vectors at several points in a body
- Generally, stress trajectories for the maximum and minimum principal stresses are drawn, and a change in trend means a change in orientation of these principal stresses.
- Collectively, principal stress trajectories are used to represent the *orientation of the* ***stress field*** *in a body*.
- In some cases the magnitude of a particular stress vector is represented by varying the spacing between the trajectories.



F I G U R E 3 . 1 3 (a) Theoretical stress trajectories of σ_1 (full lines) and σ_3 (dashed lines) in a block that is pushed from the left resisted by frictional forces at its base. Using the predicted angle between maximum principal stress (σ_1) and fault surface of around 30° (Coulomb failure criterion; Chapter 6) we can predict the orientation of faults, as shown in (b).

STRESS FIELDS

- If the stress at each point in the field is the same in magnitude and orientation, the stress field is *homogeneous;* otherwise it is *heterogeneous,* as in Figure 3.13.

- ***Homogeneity and heterogeneity of the stress field* should not be confused with *isotropic and anisotropic stress*.**

  - *Isotropic means that the principal stresses are equal* (describing a sphere), but their magnitude can vary, whereas *homogeneous stress implies that the orientation and shape of the stress ellipsoids are equal throughout the body*.

  - Therefore, in a homogeneous stress field, all principal stresses have the same orientation and magnitude.

  - *The orientation of stress trajectories under natural conditions typically is heterogeneous, or varies, arising from the presence of discontinuities in rocks, the complex interplay of more than one stress field (like gravity), or from physical variations in the composition or arrangement of material*

PetroWiki

Subsurface stress and pore pressure

petrowiki.org/Subsurface_stress_and_pore_pressure

10-point summary from PetroWiki.org

1. Forces in the Earth are quantified by means of a stress tensor, in which the individual components are tractions (with dimensions of force per unit area) acting perpendicular or parallel to three planes that are in turn orthogonal to each other.
2. The normals to the three orthogonal planes define a Cartesian coordinate system (x_1, x_2, and x_3)
3. The stress tensor has nine components, each of which has an orientation and a magnitude
4. Three of these components are normal stresses, in which the force is applied perpendicular to the plane (acting normal to a plane perpendicular to a principal axis and the other six are shear stresses, in which the force is applied along the plane in a particular direction and therefore perpendicular to principal axis
5. *In all cases, $S_{ij} = S_{ji}$, which reduces the number of independent stress components to six.*
6. At each point there is a particular stress axes orientation for which all shear stress components are zero, the directions of which are referred to as the “principal stress directions.”

PetroWiki

Subsurface stress and pore pressure

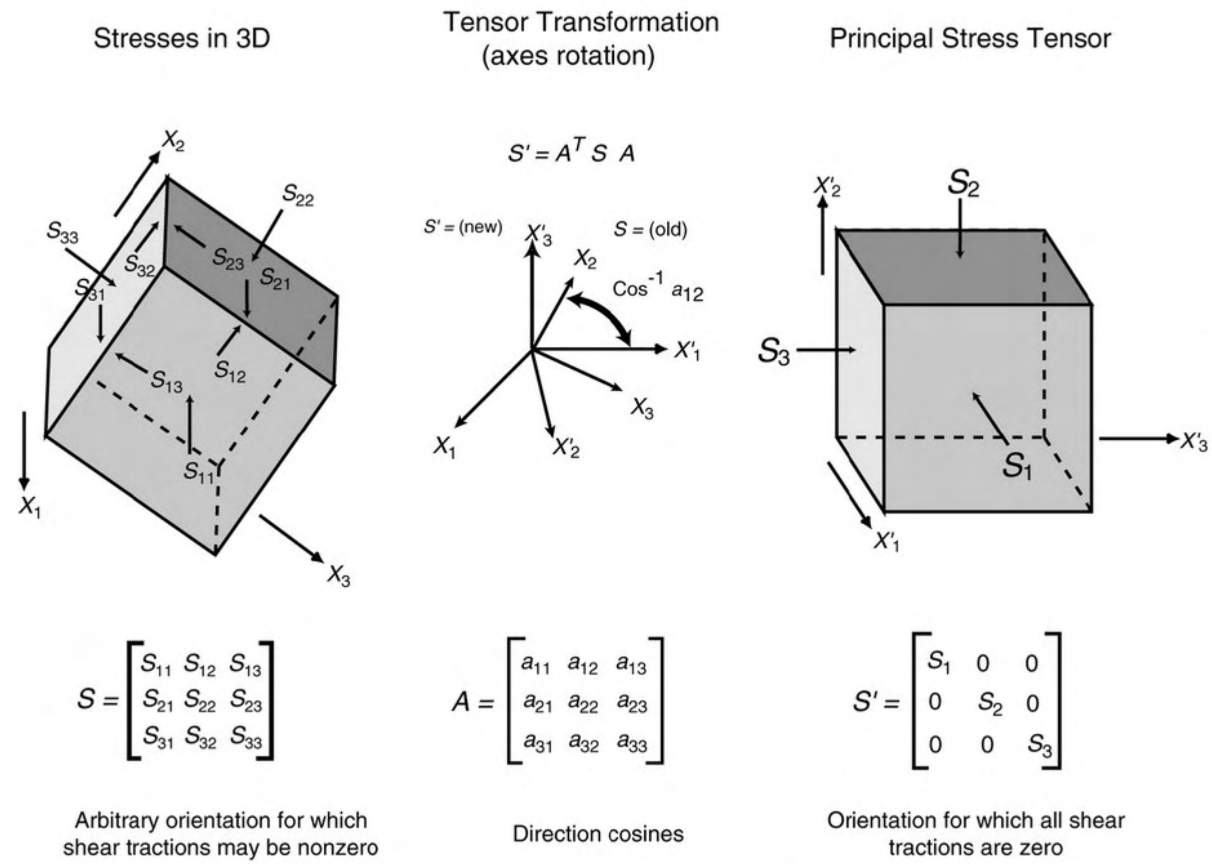
petrowiki.org/Subsurface_stress_and_pore_pressure

10-point summary from PetroWiki.org (continued from previous page)

7. The stresses acting along the principal stress axes are called principal stresses.

8. The magnitudes of the principal stresses are S_1, S_2, and S_3, corresponding to the greatest principal stress, the intermediate principal stress, and the least principal stress, respectively.

9. Coordinate transformations between the principal stress tensor and any other arbitrarily oriented stress tensor are accomplished through tensor rotations.



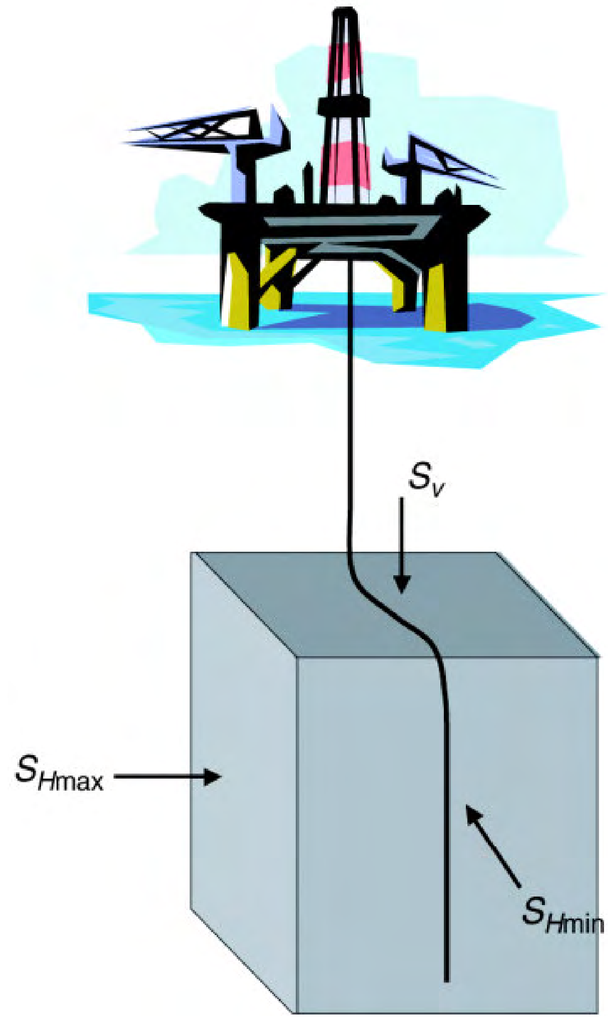
petrowiki.org/images/1/18/Devol2_1102final_Page_002_Image_0001.png

10-point summary from PetroWiki.org (continued from previous page)

10. It has been found in most parts of the world, at depths within reach of the drill bit, that the stress acting vertically on a horizontal plane (defined as the vertical stress, S_v) is a principal stress.

This requires that the other two principal stresses act in a horizontal direction.

Because these horizontal stresses almost always have different magnitudes, they are referred to as the greatest horizontal stress, S_{Hmax}, and the least horizontal stress, S_{Hmin}



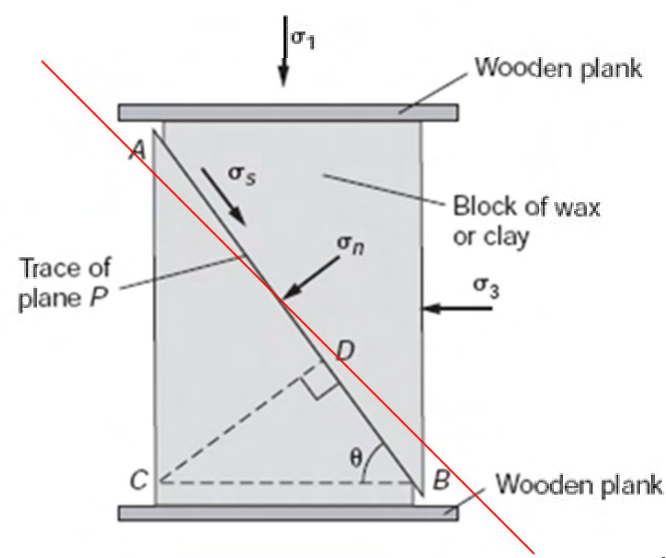
METHODS OF STRESS MEASUREMENT

TABLE 3.3 SOME STRESS MEASUREMENT METHODS

Borehole breakouts	The shape of a borehole changes after drilling in response to stresses in the host rock. Specifically, the hole becomes elliptical with the long axis of the ellipse parallel to minimum horizontal principal stress ($\sigma_{s,\,hor}$).
Hydrofracture	If water is pumped under sufficient pressure into a well that is sealed off, the host rock will fracture. These fractures will be parallel to the maximum principal stress (σ_1), because the water pressure necessary to open the fractures is equal to the minimum principal stress.
Strain release	A strain gauge, consisting of tiny electrical resistors in a thin plastic sheet, is glued to the bottom of a borehole. The hole is drilled deeper with a hollow drill bit (called *overcoring*), thereby separating the core to which the strain gauge is connected from the wall of the hole. The inner core expands (by elastic relaxation), which is measured by the strain gauge. The direction of maximum elongation is parallel to the direction of maximum compressive stress and its magnitude is proportional to stress according to Hooke's Law (see Chapter 5).
Fault-plane solutions	When an earthquake occurs, records of the first motion on seismographs around the world enable us to divide the world into two sectors of compression and two sectors of tension. These zones are separated by the orientation of two perpendicular planes. One of these planes is the fault plane on which the earthquake occurred, and from the distribution of compressive and tensile sectors, the sense of slip on the fault can be determined. Seismologists assume that the bisector of the two planes in the tensile sector represents the minimum principal stress (σ_3) and the bisector in the compressive field is taken to be parallel to the maximum compressive stress (σ_1).

METHODS OF STRESS MEASUREMENT

- Present-day stress determinations, like borehole measurements, give differential stress magnitudes that likewise range from tens to hundreds of megapascals.
- Realize, though, that these methods only record stress magnitudes in the outermost part (upper crust) of Earth



45° dipping plane

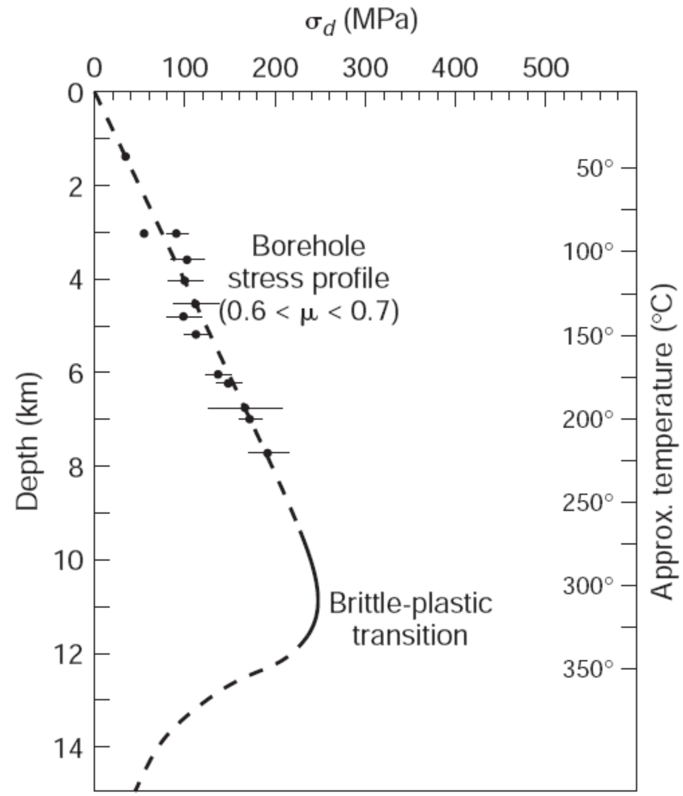
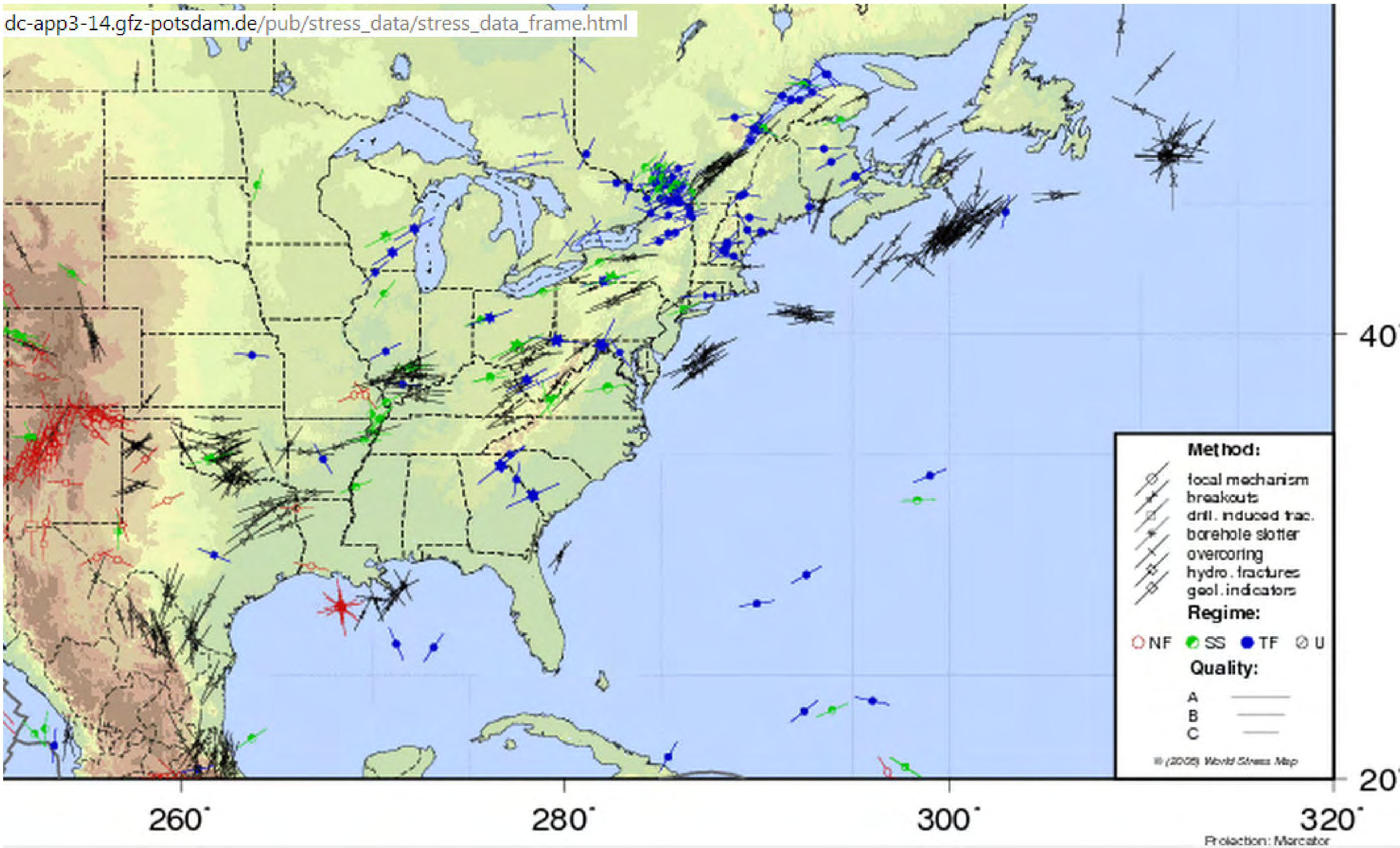


FIGURE 3.14 In situ borehole measurements of differential stress (σ_d) with depth, indicating a friction coefficient (μ) in the range of 0.6–0.7 for the upper crust.



dc-app3-14.gfz-potsdam.de/pub/stress_data/stress_data_frame.html
Method:
focal mechanism
breakouts
drill. induced frac.
borehole slotter
overcoring
hydro. fractures
geol. indicators
Regime:
NF SS TF U
Quality:
A
B
C
40°
20°
260°
280°
300°
320°
Projection: Mercator

PRESENT-DAY STRESS

From large data sets of present-day stress measurements we find that the results are generally in good agreement about the orientation of the principal stresses and that they compare reasonably well in magnitude.

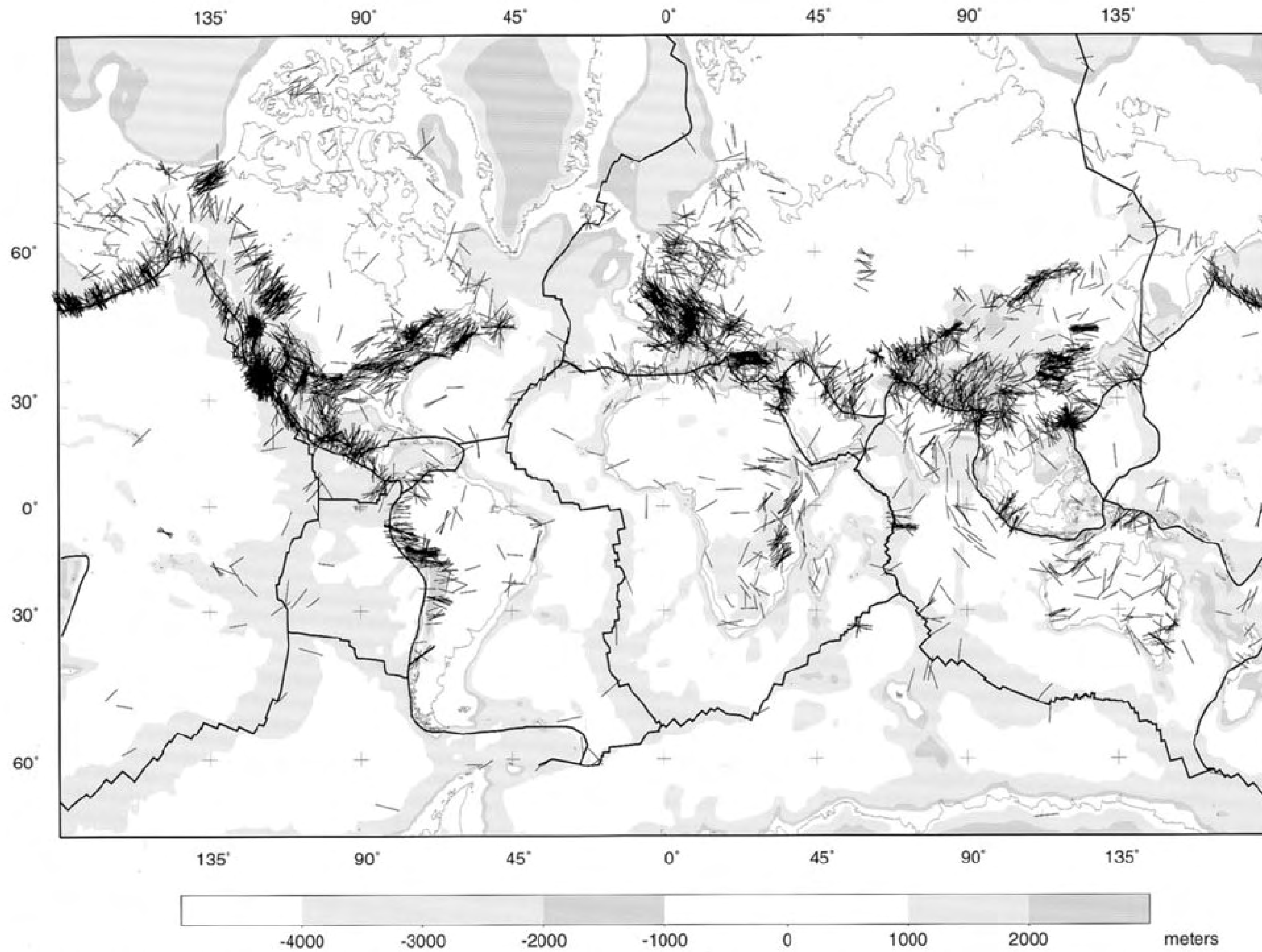


FIGURE 3.15 (a) World Stress Map showing orientations of the maximum horizontal stress superimposed on topography.

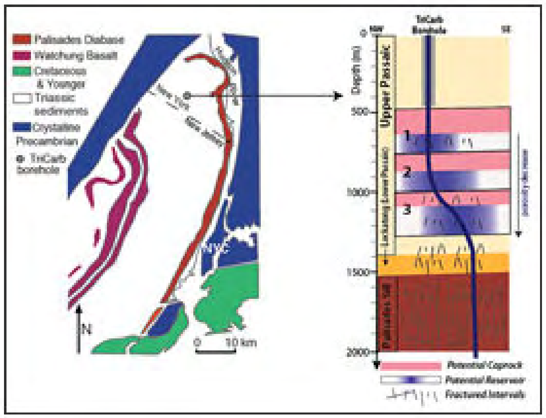
STRESS IN THE EARTH'S CRUST

www.ldeo.columbia.edu/BRG/research_projects/carbon/#carbon5

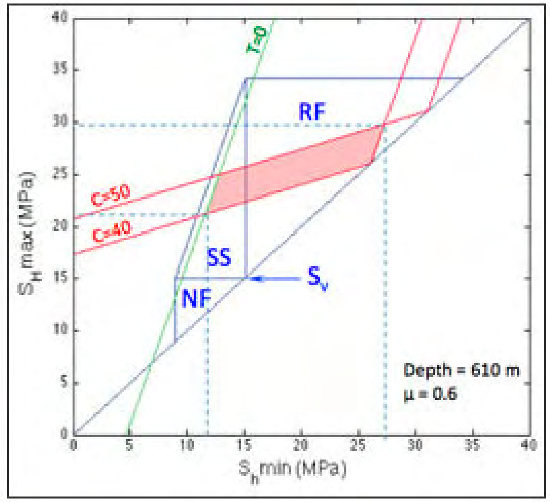
In Situ Stress and Risk of Induced Seismicity from CO_2 Injection

Natalia V. Zakharova and David S. Goldberg

Induced seismicity due to pore pressure increase presents a significant risk for carbon sequestration in fractured formations. In order to evaluate fracture stability, detailed knowledge of the in situ stress is required. High-resolution wellbore images allow identifying both natural discontinuities and drilling-induced failures indicative of the in situ stress regime. This study demonstrates an application of borehole techniques for stress analysis at a potential CO_2-storage site in the Newark Rift basin in the northeastern U.S (TriCarb project). Effects of borehole deviation on wellbore failure and constraints on complete stress field have been determined. Stability of natural fractures at various depths was evaluated for a range of potential stress profiles. Preliminary analysis suggests that a significant capacity for pore pressure increase without fracture reactivation exists in deeper reservoirs, but additional in situ test data are needed for a more complete assessment of the induced seismic risk from potential CO_2 injection in the region.



Study site location, and a schematic diagram of the TriCarb borehole indicating three potential reservoir-caprock pairs in the Passaic formation of the Newark Basin supergroup.



Magnitudes of horizontal stresses at the top of reservoir layer 1 based on Coulomb faulting theory and Anderson's classification of faults.

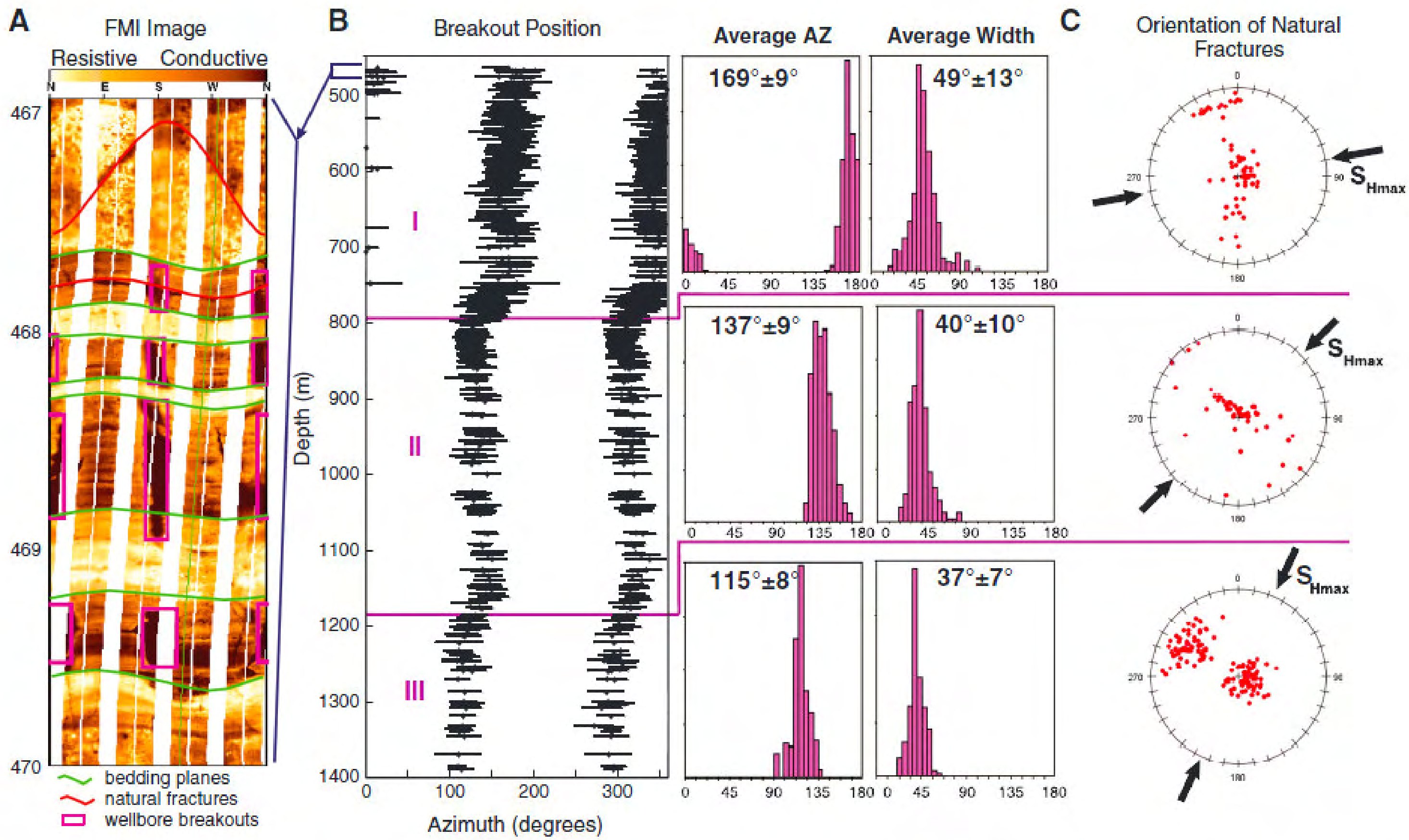


Figure 3. Wellbore breakouts and natural fractures in the TriCarb well: (a) a representative section of interpreted FMI™ image; (b) breakouts position and width as a function of depth, histograms summarize breakouts statistics for three depth intervals with distinctly different breakouts orientation (Zones I-III); and (c) orientation of natural fractures for the same three intervals (red dots represent poles to fracture planes in the equal-angle low-hemisphere projection), black arrows indicate the maximum horizontal stress orientation inferred from breakout observation (Prepared using GMI™ software).

AGU Journal of Geophysical Research: Solid Earth 10.1002/2013JB010492

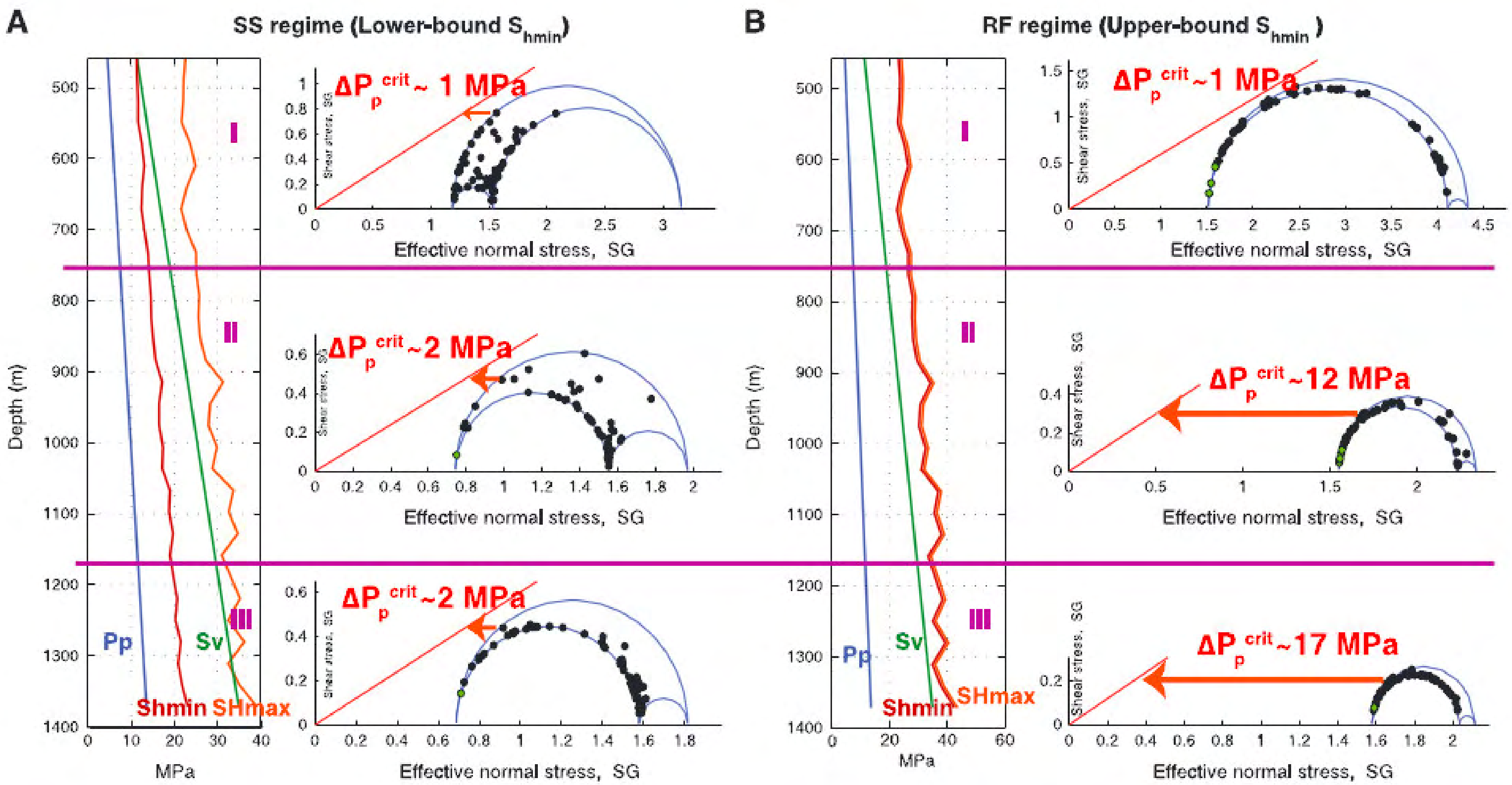


Figure 6. Stability of natural fractures for two end-member stress profiles in the TriCarb well: (a) low-bound S_{hmin}, strike-slip regime; (b) upper bound S_{hmin}, reverse-faulting regime (prepared using GMI™ software). Three-dimensional Mohr diagrams represent effective stresses on fracture planes identified in FMI™ images for the three depth zones under ambient conditions (hydrostatic pore pressure). The red lines indicate Coulomb failure limit for each zone; a critical pore pressure increase required to reach the failure limit in each zone is labeled in red.

High horizontal compressive stresses indicate a reverse faulting/strike-slip stress regime, consistent with regional stress information from previous studies

- At this site, the minimum horizontal stress magnitude varies from 10 to 25MPa at ~450m to 20 to 50MPa at ~1350 m.

- An important trend in the relative stress magnitudes versus depth—horizontal stresses decrease steadily with respect to the vertical stress—resulting in a gradual transition from a reverse faulting (RF) to reverse faulting/strike-slip (RF/SS) stress regime.

- In both scenarios, the shallow interval at 500m depth is critically stressed, and therefore, a small change in the effective stress potentially caused by elevated pore pressure would induce failure on favorably oriented fractures or faults.

- The deeper intervals in this well, however, could potentially accommodate a significant pore pressure increase (of 1–2MPa).

PRESENT-DAY STRESS

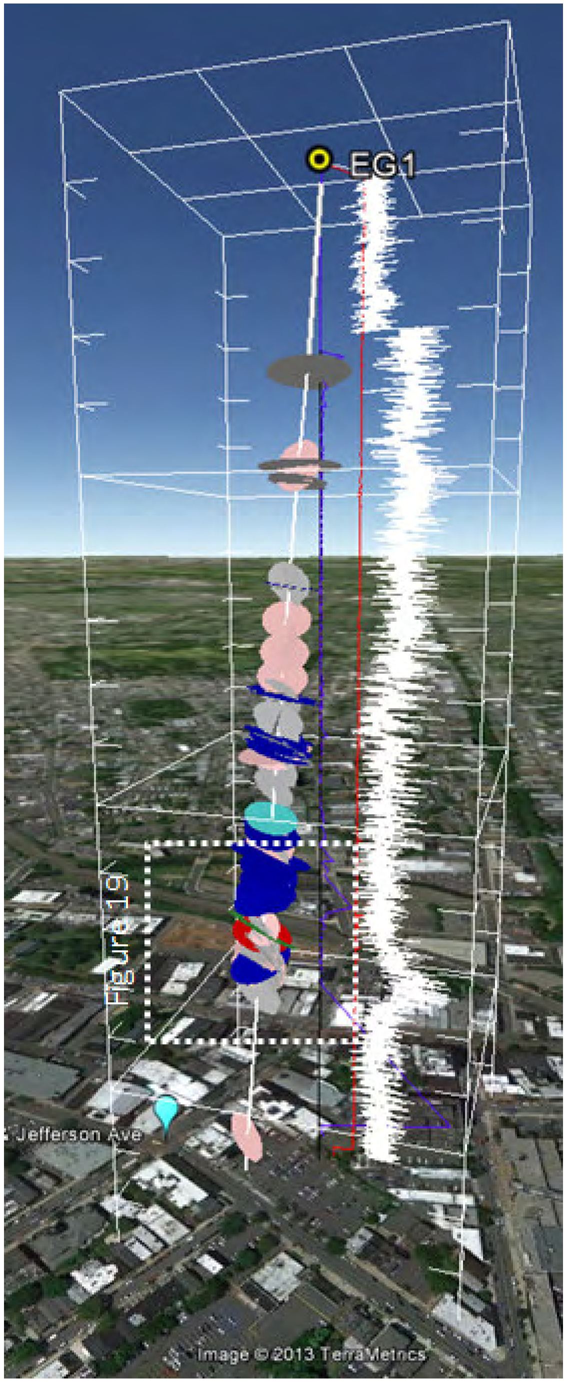
- In geology, we are commonly faced with situations where we need to ask, if the pressure increases or decreases in an area, or within a well, what would be the consequences of this pressure change, and under what conditions does the substrate rupture, or ‘fail’.

- For example, underground mines release pressure when tunnels are excavated in the subsurface.

- If we know the local stress at a place in the crust, or the **in-situ stress field**, and the orientation of naturally occurring discontinuities (like fractures or “joints”), we can predict what planes are likely to fail and slip, given knowledge of the type of expected disturbance and the mechanical properties of the rock.

- Similarly, observed ruptures, such as “roof spalls” in mines gives us clues as to the orientation of the *horizontal principal components of the crustal stress field.*

- *But in order to work with stress in deterministic studies, we need to understand how to quantify the results of applying stress on crustal materials, that is,* ***STRAIN****, and how strains accumulate from both brittle and plastic strain processes.*


EG1
Figure 19
Jefferson Ave
Image © 2013 TerraMetrics

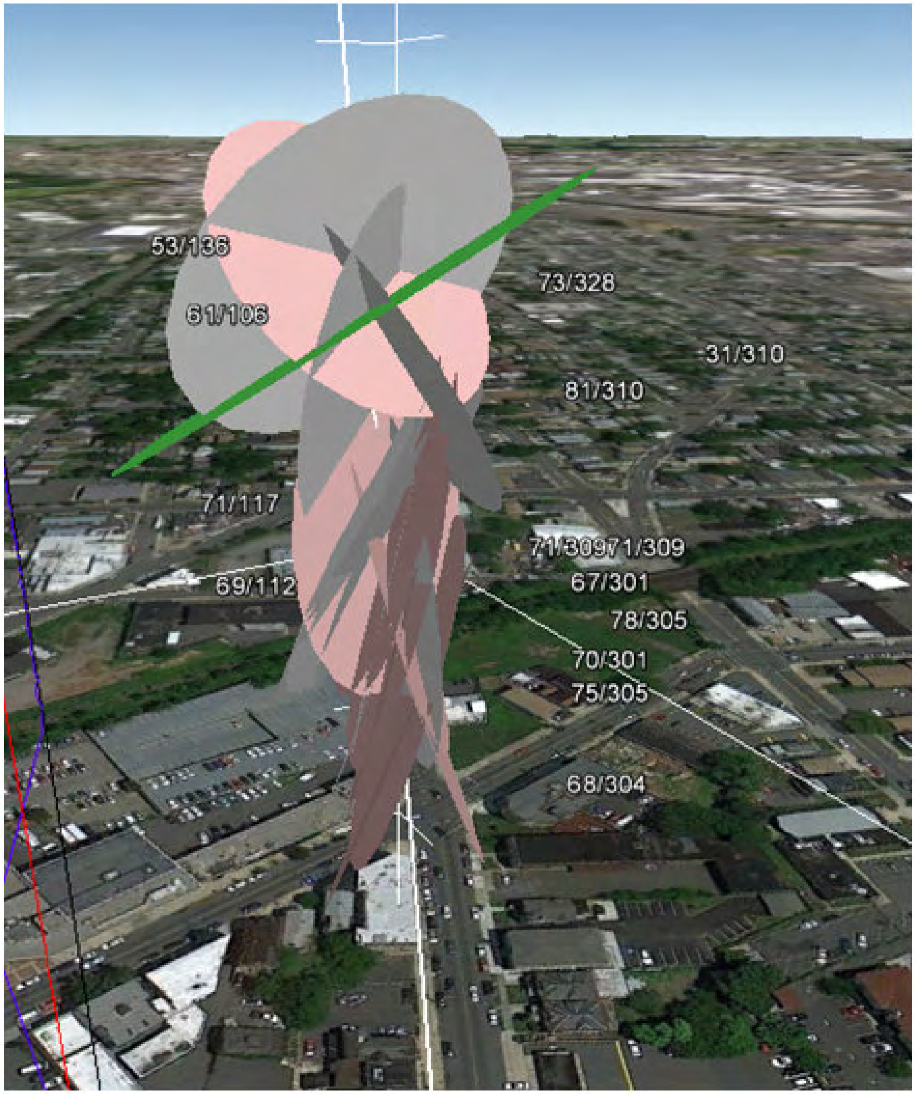

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Figure 19

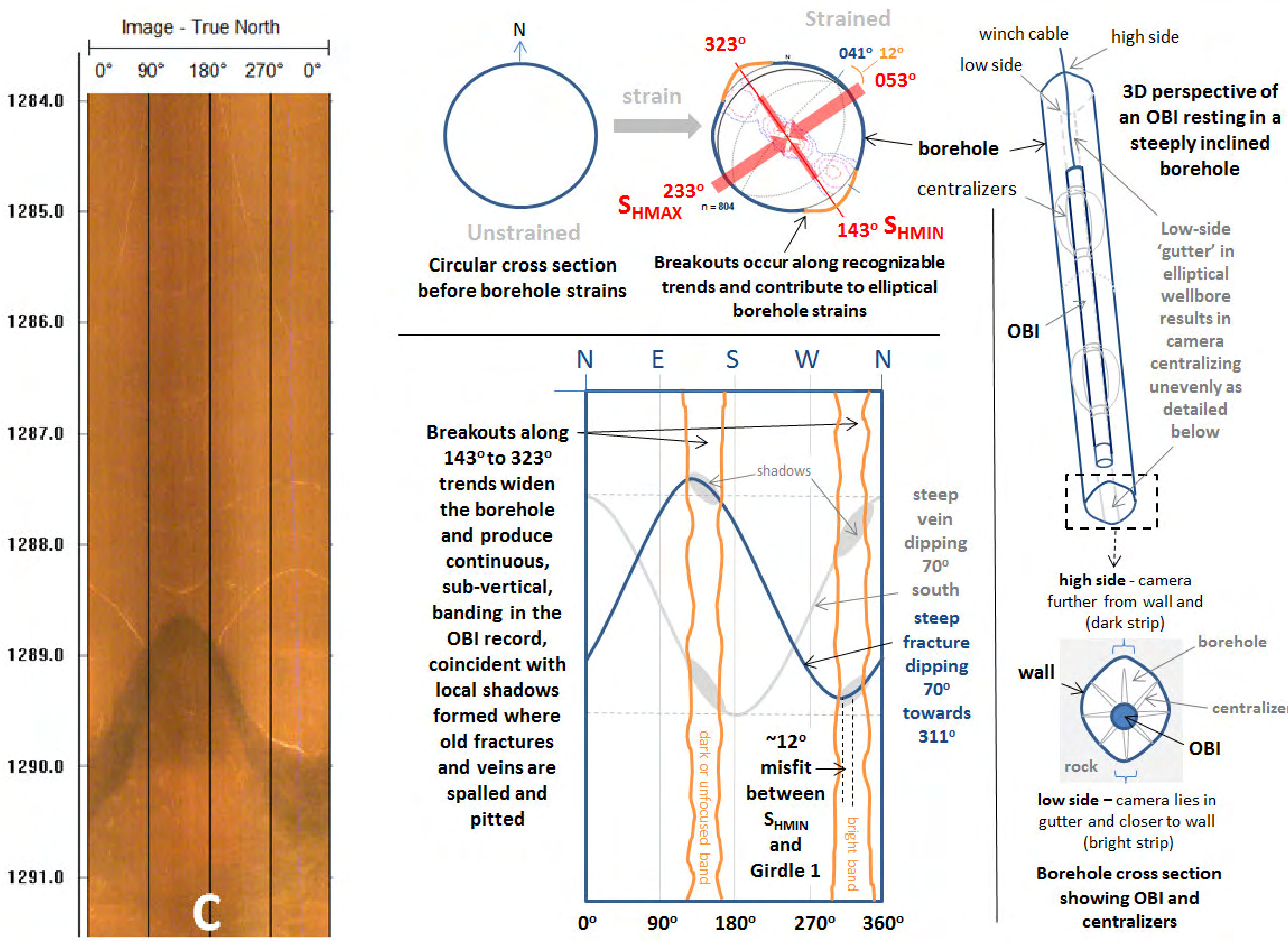
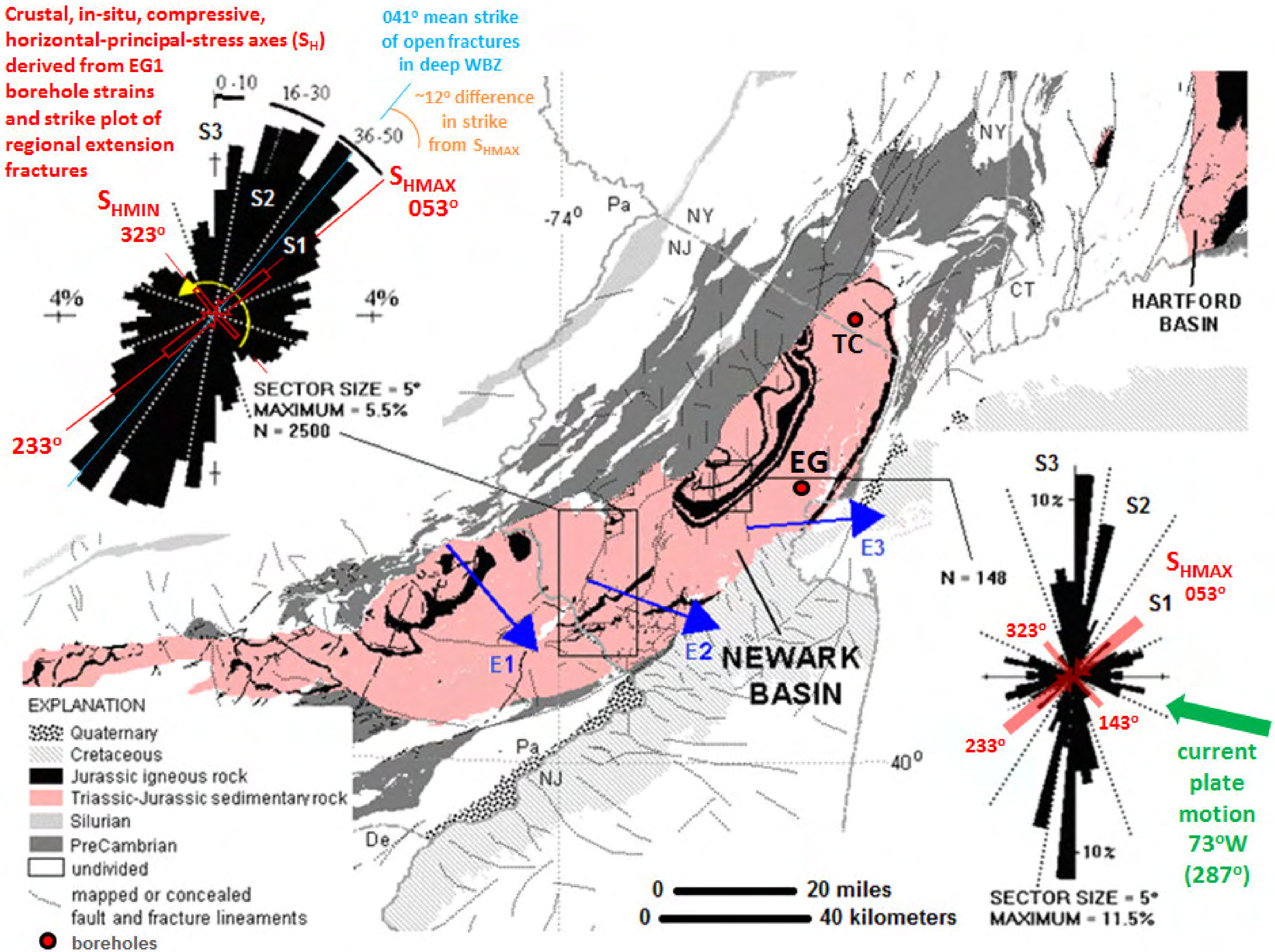


Image - True North
0° 90° 180° 270° 0°
1284.0
1285.0
1286.0
1287.0
1288.0
1289.0
1290.0
1291.0
C
N
strain
Unstrained
Circular cross section before borehole strains
Strained
323°
041° 12°
053°
233°
SHMAX
n = 804
143° SHMIN
borehole
Breakouts occur along recognizable trends and contribute to elliptical borehole strains
N E S W N
Breakouts along 143° to 323° trends widen the borehole and produce continuous, sub-vertical, banding in the OBI record, coincident with local shadows formed where old fractures and veins are spalled and pitted
shadows
steep vein dipping 70° south
steep fracture dipping 70° towards 311°
dark or unfocused band
~12° misfit between SHMIN and Girdle 1
bright band
0° 90° 180° 270° 360°
winch cable
high side
low side
3D perspective of an OBI resting in a steeply inclined borehole
centralizers
OBI
Low-side 'gutter' in elliptical wellbore results in camera centralizing unevenly as detailed below
high side - camera further from wall and (dark strip)
borehole
wall
centralizer
OBI
rock
low side – camera lies in gutter and closer to wall (bright strip)
Borehole cross section showing OBI and centralizers


Crustal, in-situ, compressive, horizontal-principal-stress axes (S_H) derived from EG1 borehole strains and strike plot of regional extension fractures
041° mean strike of open fractures in deep WBZ
~12° difference in strike from S_{HMAX}
0-10
16-30
36-50
S3
S2
S1
S_{HMAX} 053°
S_{HMIN} 323°
233°
4%
4%
SECTOR SIZE = 5°
MAXIMUM = 5.5%
N = 2500
-74°
Pa
NY
NJ
CT
HARTFORD BASIN
TC
EG
E1
E2
E3
NEWARK BASIN
N = 148
S3
S2
S1
10%
S_{HMAX} 053°
323°
143°
233°
current plate motion 73°W (287°)
10%
SECTOR SIZE = 5°
MAXIMUM = 11.5%
40°
Pa
NJ
De
0 20 miles
0 40 kilometers
EXPLANATION
Quaternary
Cretaceous
Jurassic igneous rock
Triassic-Jurassic sedimentary rock
Silurian
PreCambrian
undivided
mapped or concealed fault and fracture lineaments
boreholes

PRESENT-DAY STRESS AND PLATE-DRIVING FORCES

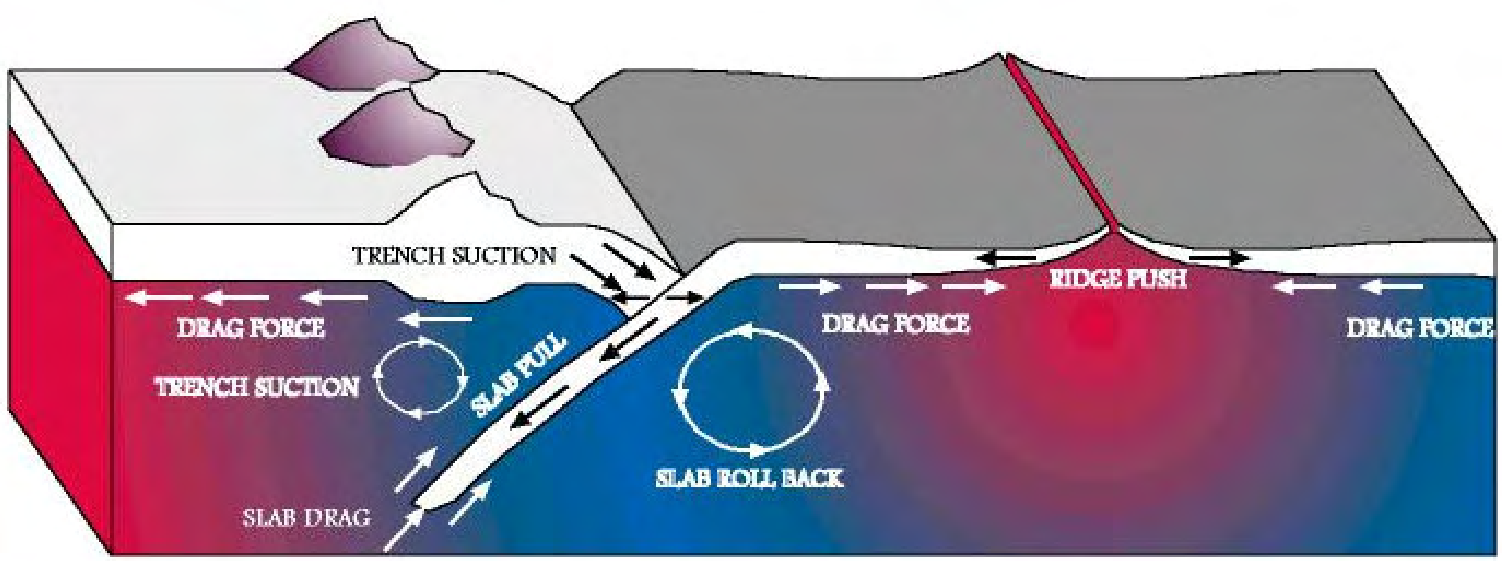


Figure 1: Basic schematic of different Plate Driving Forces. www.umich.edu/~gs265/tecpaper.htm